International Macroeconomics[1]

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Chapter 1

Global Imbalances

1.1 Balance-of-Payments Accounting

A country's international transactions are recorded in the balance-of-payments accounts. In the United States, the balance-of-payments accounts are compiled by the Bureau of Economic Analysis (BEA), which belongs to the U.S. Department of Commerce. Up-to-date balance of payments data can be found on the BEA's website at `http://www.bea.gov`.

A country's balance of payments has two main components: the current account and the financial account. The current account records exports and imports of goods and services and international receipts or payments of income. Exports and income receipts enter with a plus and imports and income payments enter with a minus. For example, if a U.S. resident buys a smartphone from South Korea for \$500, then the U.S. current account goes down by \$500. This is because this transaction represents an import of goods worth \$500.

The financial account keeps record of sales of assets to foreigners and purchases of assets located abroad. Thus, the financial account measures changes in a country's net foreign asset position. Sales of assets to foreigners are given a positive sign and purchases of assets located abroad a negative sign. For example, in the case of the import of the smartphone, if the U.S. resident pays with U.S. currency, then a South Korean resident (Samsung) is buying U.S. assets (currency) for \$500, so the U.S. financial account receives a positive entry of \$500.[1]

The smartphone example illustrates a fundamental principle of balance-of-payments accounting known as *double-entry bookkeeping*. Each transaction enters the balance of payments twice, once with a positive sign and once with a negative sign. To illustrate this principle with another example, suppose that an Italian friend of yours comes to visit you in New York and stays at the Lucerne Hotel. He pays \$400 for his lodging with his Italian VISA card. In this case, the U.S. is exporting a service (hotel accommodation), so the current account increases by \$400. At the same time, the Lucerne Hotel purchases a financial asset worth \$400 (the promise of VISA-Italy to

[1] There is a third component of the Balance of Payments called the capital account. This component is quantitatively insignificant in the United States, so we will ignore it. It keeps record of international transfers of financial capital. The major types of entries in the capital account are debt forgiveness and migrants' transfers (goods and financial assets accompanying migrants as they leave or enter the country). Although insignificant in the United States, movements in the capital account can be important in other countries. For instance, in July 2007 the U.S. Treasury Department announced that the United States, Germany, and Russia will provide debt relief to Afghanistan for more than 11 billion dollars. This is a significant amount for the balance of payments accounts of Afghanistan, representing about 99 percent of its foreign debt obligations. But the amount involved in this debt relief operation is a small figure for the balance of payments of the three donor countries. The capital account also records payments associated with foreign insurance contracts. For example, in the fourth quarter of 2012 net capital account receipts were \$7.2 billion reflecting receipts from foreign insurance companies for losses resulting from Hurricane Sandy.

pay \$400), which decreases the U.S. financial account by \$400.[2]

An implication of the double-entry bookkeeping methodology is that any change in the current account must be reflected in an equivalent change in the country's financial account, that is, the current account equals the difference between a country's purchases of assets from foreigners and its sales of assets to them, which is the financial account preceded by a minus sign. This relationship is known as the **fundamental balance-of-payments identity**. Formally,

$$\text{Current Account Balance} = -\text{ Financial Account Balance.}$$

A more detailed decomposition of the balance-of-payments accounts is as follows:

1. **Current Account:** net exports (i.e., difference between exports and imports) of goods and services and net international income receipts.
    (a) **Trade Balance (or Balance on Goods and Services):** difference between exports and imports of goods and services.
        i. **Merchandise Trade Balance (or Balance on Goods):** net exports of goods.
        ii. **Services Balance:** Net receipts from items such as transportation, travel expenditures, and legal assistance.
    (b) **Income Balance:**
        i. **Net investment income:** Difference between income receipts on U.S.-owned assets abroad and income payments on foreign-owned assets in the United States. Includes international interest and dividend payments and earnings of domestically owned firms operating abroad.
        ii. **Net international compensation to employees** This account measures U.S. compensation receipts from (1) earnings

[2] How does this transaction affect the Italian balance of payments accounts?

of U.S. residents employed temporarily abroad, (2) earnings of U.S. residents employed by foreign governments in the United States, and (3) earnings of U.S. residents employed by international organizations in the United States, which is the largest of the three categories. This account also measures U.S. compensation payments to (1) Canadian and Mexican workers who commute to work in the United States, (2) foreign students studying at colleges and universities in the United States, (3) foreign professionals temporarily residing in the United States, (4) foreign temporary agricultural workers in the United States, and (5) foreign temporary nonagricultural workers in the United States. The largest categories of compensation payments are payments to foreign temporary agricultural workers and to foreign temporary nonagricultural workers.

(c) **Net Unilateral Transfers:** Difference between gifts (that is, payments that do not correspond to purchases of any good, service, or asset) received from the rest of the world and gifts made by the United States to foreign countries. Over the past decade private remittances have become a major component of Net Unilateral Transfers. For example, payments by a Mexican citizen residing in the United States to relatives in Mexico would enter with a minus in the current account as they represent a payment of someone residing in the U.S. to someone residing abroad. This account also includes U.S. Government Grants which provide U.S. government financing to transfer real resources or financial assets to foreigners under programs enacted by the U.S. Congress for the provision of nonmilitary and military foreign assistance (grants) for which no repayment is expected.

2. **Financial Account:** Difference between sales of assets to foreigners and purchases of assets from foreigners.

   (a) U.S.-owned assets abroad consist of:

      i. U.S. purchases and sales of foreign securities, U.S. bank lending to foreigners, and U.S. direct investment abroad.

   (b) Foreign-owned assets held in the United States consist of:

      i. Foreign purchases and sales of U.S. securities, U.S. bank borrowing from foreigners, and foreign direct investment in the United States.

Transactions for financial derivatives are also recorded in the financial account.

The components of the current account are linked by the accounting identity

$$
\begin{aligned}
\text{Current Account Balance} &= \text{Trade Balance} \\
&+ \text{Income Balance} \\
&+ \text{Net Unilateral Transfers.}
\end{aligned}
$$

And the components of the trade balance satisfy

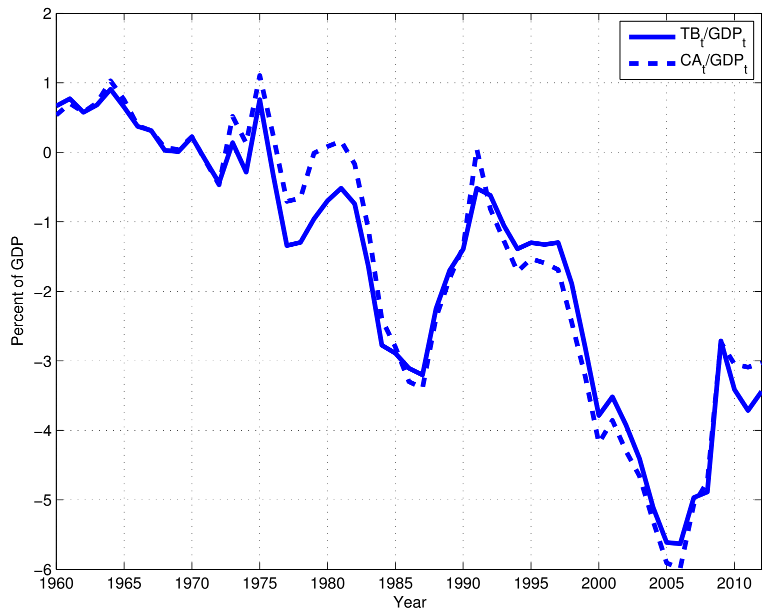
$$
\begin{aligned}
\text{Trade Balance} &= \text{Merchandise Trade Balance} \\
&+ \text{Services Balance}
\end{aligned}
$$

1.2 The Current Account

What does the U.S. current account look like? Take a look at table 1.1. It displays the U.S. current account for 2012. In that year, the United States experienced large deficits in both the current account and the trade balance of about half a trillion dollars, or about 3 percent of GDP. Current-account and trade-balance deficits are frequently observed. In fact, as shown in figure 1.1 the U.S. trade- and current-account balances have been in deficit for more than 30 years. Moreover, during this period the observed current-account and trade-balance deficits have been roughly equal to each other.

In 2012, the United States was a net importer of goods, with a merchandise trade deficit of 4.7% of GDP and at the same time a net exporter

Figure 1.1: The U.S. Trade Balance and Current Account As Percentages Of GDP: 1960-2012



Source: `http://www.bea.gov`

Table 1.1: U.S. Current Account, 2012.

Item	Billions of dollars	Percentage of GDP
Current Account	-475.0	-3.0
Trade Balance	-539.5	-3.4
Merchandise Trade Balance	-735.3	-4.7
Services Balance	195.8	1.2
Income Balance	198.6	1.3
Net Investment Income	206.2	1.3
Net International Compensation to Employees	-7.6	-0.0
Net Unilateral Transfers	-134.1	-0.9
Private Remittances	-77.6	-0.5
U.S. Government Transfers	-56.5	-0.4

Source: Bureau of Economic Analysis, U.S. Department of Commerce, `http://www.bea.gov`.

of services, with a service balance surplus of 1.2% of GDP. The U.S. has a comparative advantage in the production of human-capital-intensive services, such as professional consulting, higher education, research and development, and health care. At the same time, the U.S. imports basic goods, such as primary commodities, textiles, and consumer durables.

The fact that in the United States the trade balance and the current account have been broadly equal to each other in magnitude over the past thirty years means that the sum of the other two components of the current account, the income balance and net unilateral transfers, were close to zero in most years. The year 2012 was a little bit atypical in that regard. The income balance showed a surplus of $198 billion and net unilateral transfers showed a deficit of $134 billion, accounting for the $65 difference between the current account and the trade balance.

In 2012, as in prior years, the United States made more gifts to other nations than it received. About 60 percent of these gifts are remittances of foreign workers residing in the U.S. to relatives in their countries of origin. Typically foreign workers residing in the U.S. send much larger remittances abroad than U.S. workers residing abroad send back to the United States. For example, in 2009 personal transfers of U.S. immigrants to foreign residents were \$ 37,552 million (this would enter with a minus in the BOP) but personal transfers from U.S. emigrants living abroad to U.S. residents were only \$ 766 million (those would enter with a minus sign in the BOP accounts). That is net private remittances were almost the same as gross remittances. Overall, net remittances is a small fraction of the U.S. balance of payments. But, for some countries, net receipts of remittances can represent a substantial source of foreign income. For example, in 2004 Mexico received about 2.5 percent of GDP in net remittances. This source of income was responsible for the fact that in that year Mexico's current account deficit was smaller than its trade deficit, despite the fact that Mexico, being a net debtor to the rest of the world, had to make large international interest payments. In the United States net unilateral transfers have been negative ever since the end of World War II, with one exception. In 1991, net unilateral transfers were positive because of the payments the U.S. received from its allies in compensation for the expenses incurred during the Gulf war.

The balance on the current account may be larger or smaller than the balance on the trade account. Also, both the trade balance and the current account may be positive or negative and they need not have the same

Table 1.2: Trade Balance and Current Account as Percentages of GDP in 2005 for Selected Countries

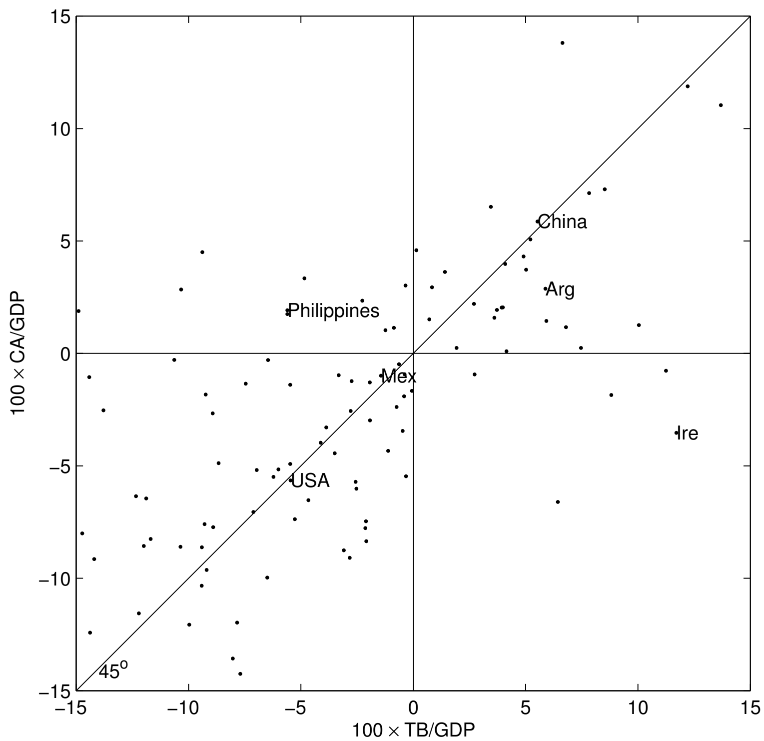
Country	TB/GDP	CA/GDP
Argentina	5.9	2.9
China	5.5	5.9
Ireland	11.7	-3.5
Mexico	-1.4	-1.0
Philippines	-5.6	1.9
United States	-5.5	-5.6

Source: World Development Indicators. Available online at `http://databank.worldbank.org`. Note: CA denotes current account, and TB denotes trade balance.

sign. Figure 1.2 illustrates this point. It displays the trade balance and the current account as percentages of GDP in 2005 (TB/GDP and CA/GDP, respectively) for 102 countries. The space (TB/GDP,CA/GDP) is divided into six regions, depending on the signs of the current account and the trade balance and on their relative magnitudes. Table 1.2 extracts six countries from this group with CA/GDP and TB/GDP pairs located in different regions.

Argentina is an example of a country that in 2005 ran trade-balance and current-account surpluses, with the trade balance exceeding the current-account. The current account surplus was smaller than the trade balance surplus because of interest payments that the country made on its external debt, which caused the income balance to be negative. Historically, Argentina's foreign interest obligations have been larger than the trade balance resulting in negative current account balances. However, in 2001, Argentina

Figure 1.2: Trade Balances and Current Account Balances Across Countries in 2005



Data Source: World Development Indicators. Note: TB denotes the trade balance in goods and services and CA denotes the current account balance. There are 102 countries included in the sample.

defaulted on much of its external debt thereby reducing its net interest payments on foreign debt.

Like Argentina, China displays both a current-account and a trade-balance surplus. However, unlike Argentina, the Chinese current-account surplus is larger than its trade-balance surplus. This difference can be explained by the fact that China, unlike Argentina, is a net creditor to the rest of the world, and thus receives positive net investment income.

The Philippines provides an example of a country with a current account surplus in spite of a sizable trade-balance deficit. The positive current account balance is the consequence of large personal remittances received (amounting to 13 percent of GDP in 2005) from overseas Filipino workers.

Mexico, the United States, and Ireland all experienced current-account deficits in 2005. In the case of Mexico and the United States, the current-account deficits were associated with trade deficits of about equal sizes. In the case of Mexico, the current-account deficit was slightly smaller than the trade deficit because of remittances received from Mexicans working in the United States. These very same remittances explain to some extent why the United States current account deficit exceeded its trade deficit.

Finally, the current-account deficit in Ireland was accompanied by a large trade surplus of about 11.7 percent of GDP. In the 1980s, Ireland embarked on a remarkable growth path that earned it the nickname 'Celtic Tiger.' This growth experience was financed largely through foreign capital inflows. Gross foreign liabilities in 2005 were about 10 times as large as one annual GDP. Foreign assets were also very large so that the net international investment position of Ireland in 2005 was 'only' -20 percent of GDP. The positive trade balance surplus of 2005 reflects mainly Ireland's effort to pay income on its large external obligations.

It is evident from figure 1.2 that most (TB/GDP, CA/GDP) pairs fall around the 45-degree line. This means that for many countries the trade balance and the current account are of the same sign and of roughly the same magnitude. This clustering around the 45-degree line suggests that for many countries, including the United States, the trade balance is the main determinant of the current account.

1.3 The Current Account and the Net International Investment Position

One reason why the concept of Current Account Balance is economically important is that it reflects a country's net borrowing needs. For example, as we saw earlier, in 2012 the United States ran a current account deficit of 475 billion dollars. To pay for this deficit, the country must have either reduced part of its international asset position or increased its international liability position or both. In this way, the current account is related to changes in a country's net international investment position. The term Net International Investment Position (NIIP) is used to refer to a country's net foreign wealth, that is, the difference between the value of foreign assets owned by the country's residents and the value of the country's assets owned by foreigners. NIIP is a stock while the current account (CA) is a flow.

The net international investment position can change for two reasons. One is deficits or surpluses in the current account, which imply, respectively, net international purchases or sales of assets. The other source of changes in the NIIP is changes in the price of the financial instruments that compose the country's international asset and liability positions. So we have that

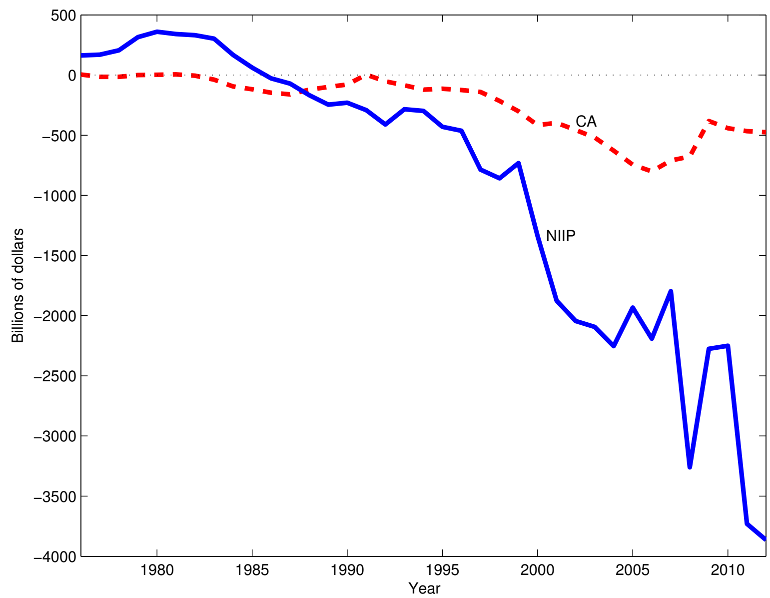
$$\Delta NIIP = CA + \text{valuation changes},$$

where the symbol Δ denotes change.

We will study the significance of price changes (or valuation changes) in the next section. In the absence of valuation changes, the *level* of the current account must equal the *change* in the net international investment position.

Figure 1.3 shows the U.S. current account balance and net international investment position since 1976. Notice that the U.S. NIIP was positive at the beginning of the sample. In the early 1980s a long sequence of current account deficits emerged that eroded the net foreign wealth of the United States. And in 1987, the nation became a net debtor to foreigners for the first time since World War I. The U.S. current account deficits did not stop in the 1990s however. By the end of that decade, the United States had become the world's largest foreign debtor. Current account deficits continued to expand for twenty five years. Only shortly before the onset of the Great Recession of 2008, did this trend stop and current account deficits became smaller in magnitude. By the end of 2012, the net international investment position of the United States stood at -3.9 trillion dollars or 25 percent of GDP. This is a big number, and many economist wonder whether the observed downward trend in the net foreign investment position is sus-

Figure 1.3: The U.S. Current Account (CA) and Net International Investment Position (NIIP)



Source: `http://www.bea.gov`

tainable over time.[3] This concern stems from the fact that countries that accumulated large external debt to GDP ratios in the past, such as many Latin American countries in the 1980s, Southeast Asian countries in the 1990s, and more recently peripheral European countries, have experienced sudden reversals in international capital flows that were followed by costly financial and economic crises. Indeed the 2008 financial meltdown in the United States has brought this issue to the fore.

1.4 Valuation Changes and the Net International Investment Position

We saw earlier that a country's net international investment position can change either because of current account surpluses or deficits or because of changes in the value of its international asset and liability positions.

To understand how valuation changes can alter a country's NIIP, consider the following hypothetical example. Suppose a country's international asset position, denoted A, consists of 25 shares in the Italian company Fiat. Suppose the price of each share in Fiat is 2 euros. Then we have that the foreign asset position measured in euros is $25 \times 2 = 50$ euro. Suppose the

[3] As dramatic as it may seem, the U.S. current account experience since the 1980s is not historically unprecedented. Throughout the 19th century the United States was a net foreign debtor country. It was only after the first World War that the U.S. became a net foreign creditor. Large and persistent current account imbalances were common in the period 1870-1914. Countries in Western Europe experienced massive capital outflows, i.e., current account surpluses, and Australia and the Americas experienced massive capital inflows, i.e., current account deficits. In fact, the capital flows observed between 1870 and 1914 were as persistent but even larger in size than the global current account imbalances observed since the 1980s. (see Chapter III, of the 2005 World Development Report published by the International Monetary Fund, Washington, April 2005).

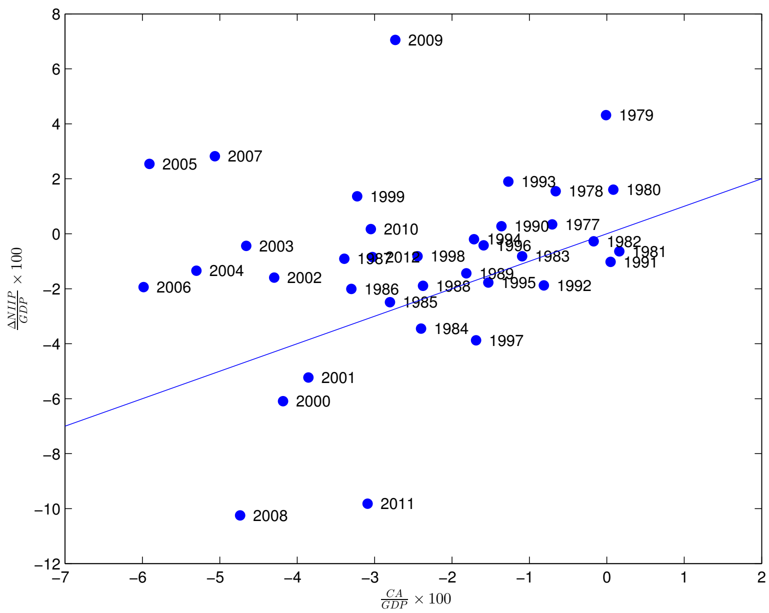
country's international liabilities, denoted L, consist of 80 units of bonds issued by the local government and held by foreigners. Suppose that the price of local bonds is \$1 per unit, where the dollar is the local currency. Then we have that total foreign liabilities are $L = 80 \times 1 = 80$ dollars. Assume further that the exchange rate is 2 dollar per euro. Then, the country's foreign asset position, measured in dollars is $A = 50 \times 2 = 100$. The country's NIIP is given by the difference between its international asset position, A, and its international liability position, L, or $NIIP = A - L = 100 - 80 = 20$. Suppose now that the euro suffers a significant depreciation, losing half of its value relative to the dollar. The new exchange rate is 1 dollar per euro. Since the country's international asset position is denominated in euros, its value in dollars automatically falls. Specifically, its new value is $A' = 50 \times 1 = 50$ dollars. The country's international liability position measured in dollars does not change, because it is composed of instruments denominated in the local currency. As a result, the new NIIP is $NIIP' = A' - L' = 50 - 80 = -30$. It follows that just because of a movement in the exchange rate, the country went from being a net creditor of the rest of the world to being a net debtor. This example illustrates that an appreciation of the domestic currency can reduce the net foreign asset position.

Consider now the effect an increase in foreign stock prices has on the net foreign asset position of the domestic country. Specifically, suppose that the price of Fiat stock jumps up to 7 euros. This price change increases the value of the country's asset position to $25 \times 7 = 175$ euros, or at an exchange rate of 1 dollars per euro to 175 dollar. The country's international liabilities do not change in value, because they do not contain shares in Fiat. The NIIP then turns positive again and equals $175 - 80 = 95$ dollars. This shows that an increase in foreign stock prices can improve a country's net international investment position.

Finally, suppose that, because of a successful fiscal reform in the domestic country, the price of local government bonds increases from 1 to 1.5 dollars. In this case, the country's gross foreign asset position remains unchanged, but its international liability position jumps up to $80 \times 1.5 = 120$ dollars. As a consequence, the NIIP falls by 40 dollars to 55 dollars. These examples show how a country's net international investment position can display large swings solely because of movements in asset prices or exchange rates.

Valuation changes have been an important source of movements in the NIIP of the United States, especially in the past two decades. Take a look at figure 1.4. It plots changes in the U.S. net international investment position as a fraction of GDP, $\frac{\Delta NIIP}{GDP}$, against the U.S. current account balance as a fraction of GDP, $\frac{CA}{GDP}$. There are 36 observations, one for each year for the period 1977 and 2012. The figure also displays with a solid line the 45-degree line. Observations for the pair $(\frac{CA}{GDP}, \frac{\Delta NIIP}{GDP})$ located below the 45-degree line correspond to years in which valuation changes were negative and observations located above the 45-degree line correspond to years in which valuation changes were positive. The figure shows that positive valuation changes have been observed more frequently than negative valuation changes. Of particular interest is the period leading to the great recession of 2008. The period 2002-2007 exhibited the largest current account deficits since 1976. In each of these years, the current account deficit exceeded 4 percent of GDP, with a cumulative deficit of 3.9 trillion dollars, or 32 percent

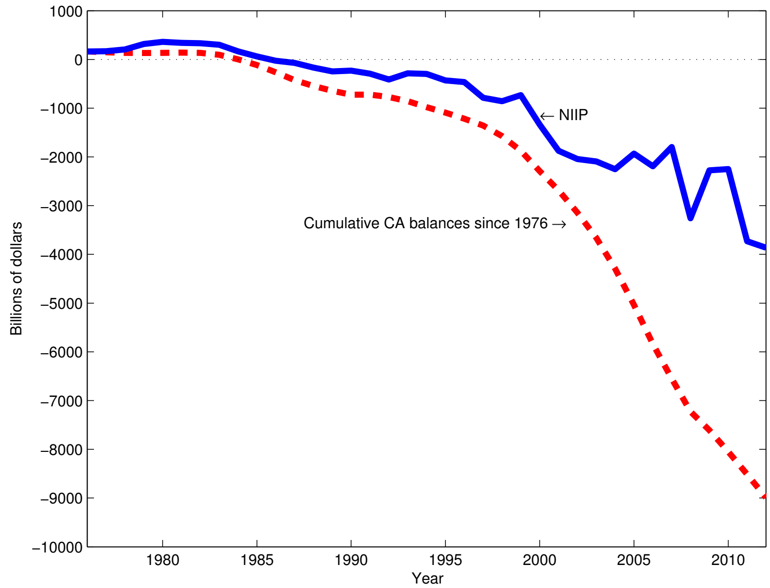
Figure 1.4: The U.S. CA and Changes in the NIIP: 1977-2012



of GDP. Nevertheless, the net international investment position actually *increased* by 0.08 trillion dollars. So in the period 2002-2007 there is a huge discrepancy of almost \$4 trillion between the accumulated current account balances and the change in the NIIP. This discrepancy is due to increases in the market value of U.S.-owned foreign assets relative to foreign-owned U.S. assets. Without this lucky strike, the U.S. net foreign asset position in 2007 would have been an external debt of about 43 percent of GDP instead of the actual 13 percent.

Another way to visualize the importance of valuation changes, is to compare the actual NIIP with the one that would have obtained in the absence of any valuation changes. Figure 1.5 plots the NIIP and the hypothetical NIIP that would have occurred if no valuation changes had taken place since

Figure 1.5: The U.S. NIIP and the Hypothetical NIIP with No Valuation Changes Since 1976



Note: the actual NIIP data are from the Bureau of Economic Analysis. The hypothetical NIIP with no valuation changes for a given year is computed as the sum of the NIIP for 1976 and the cumulative sum of current account balances from 1977 until the year in question.

1976. The hypothetical NIIP with no valuation changes for a given year is computed as the sum of the NIIP for 1976 and the cumulative sum of current account balances from 1977 until the year in question. It is clear from the graph that valuation changes became a predominant determinant of the NIIP around 2002.

What then caused the large change in the value of assets in favor of the United States over the period 2002 and 2007? Milesi-Ferretti, of the International Monetary Fund, decomposes this valuation change.[4] Because

[4]See Gian Maria Milesi-Ferretti, "A $2 Trillion Question," VOX, January 28, 2009,

valuation changes were positive during the period 2002-2007, U.S.-owned assets abroad, which are mostly denominated in foreign currency, must have increased in value by much more than foreign-owned U.S. assets, which are mostly denominated in U.S. dollars. The factors behind these asymmetric changes in value are twofold: First, the U.S. dollar depreciated relative to other currencies by about 20 percent in real terms. A depreciation of the U.S. dollar increases the dollar value of foreign-currency denominated U.S.-owned assets, while leaving unchanged the dollar value of dollar-denominated foreign-owned assets, thereby strengthening the U.S. NIIP. Second, the stock markets in foreign countries significantly outperformed the U.S. stock market. Specifically, a dollar invested in foreign stock markets in 2002 returned 2.90 dollars by the end of 2007. By contrast, a dollar invested in the U.S. market in 2002, yielded only 1.90 dollars at the end of 2007. These gains in foreign equity resulted in an increase in the net equity position of the U.S. from an insignificant level in 2002 of below \$0.04 trillion to \$3 trillion by 2007.

The large valuation changes observed in the period 2002-2007, which allowed the United States to run unprecedented current account deficits without a concomitant deterioration of its net international investment position, came to an abrupt end in 2008. Look at the dot corresponding to 2008 in figure 1.4. Notice that it is significantly below the 45-degree line, which indicates that in the year the NIIP of the United States suffered a large valuation loss. The source of this drop in value was primarily the stock market. In 2008 stock markets around the world plummeted. Because the

available online at `http://www.voxeu.org`.

net equity position of the U.S. had gotten so large by the beginning of 2008 the decline in stock prices outside of the U.S. inflicted large losses on the value of the U.S. equity portfolio.

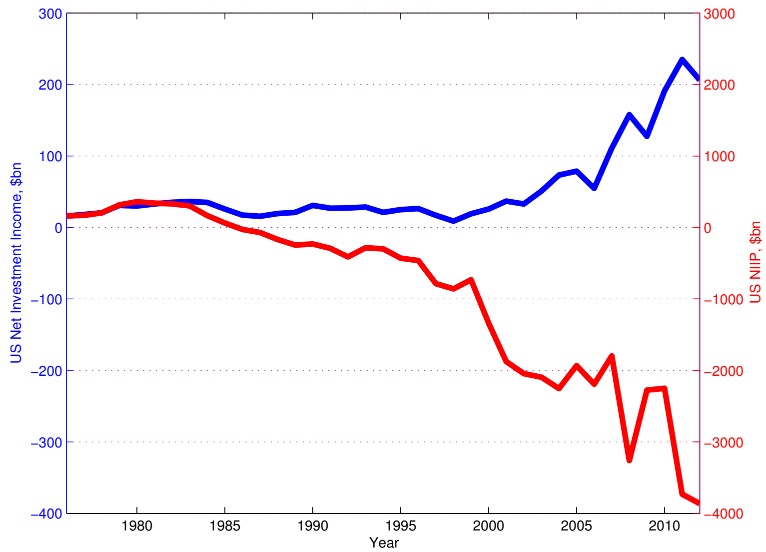
1.5 The Negative-NIIP-Positive-NII Paradox: Dark Matter?

We have documented that for the past quarter century, the United States has had a negative net international investment position ($NIIP < 0$). This means that the United States has been a net debtor to the rest of the world. One would therefore expect that during this period the U.S. paid more interest and dividends to the rest of the world than it received. In other words, we would expect that the net investment income component of the current account be negative ($NII < 0$). This is, however, not observed in the data. Take a look at figure 1.6. It shows net investment income and the net international investment position since 1976. NII is positive throughout the sample, whereas NIIP has been negative since 1986. How could it be that a debtor country, instead of having to make payments on its debt, receives income on it? Here are two explanations.

1.5.1 Dark Matter

One explanation of this paradox, proposed by Ricardo Hausmann and Federico Sturzenegger, is that the Bureau of Economic Analysis may underes-

Figure 1.6: Net Investment Income and the Net International Investment Position (United States 1976-2012)



Data Source: `http://www.bea.gov`

timate the net foreign asset holdings of the United States.[5] One source of underestimation could be that U.S. foreign direct investment contains intangible human capital, such as entrepreneurial capital and brand capital, whose value is not correctly reflected in the official balance-of-payments. At the same time, the argument goes, this human capital invested abroad may generate income for the U.S., which may be appropriately recorded. It thus becomes possible that the U.S. could display a negative net foreign asset position and at the same time positive net investment income. Hausmann and Sturzenegger refer to the unrecorded U.S. owned foreign assets as dark matter.

To illustrate the dark matter argument, suppose that McDonald's opens a restaurant in Moscow. The balance of payments will show an increase in the U.S. foreign asset position equivalent to the amount McDonald's invested in land, structures, equipment, furniture, etc. However, the market value of this investment may exceed the actual amount of dollars invested. The reason is that the brand McDonald's provides extra value to the goods (burgers) the restaurant produces. It follows that in this case the balance of payments, by not taking into account the intangible brand component of McDonald's foreign direct investment, would underestimate the U.S. international asset position. On the other hand, the profits generated by the Moscow branch of McDonald's are observable and recorded, so they make their way into the income account of the balance of payments.

According to the Hausmann-Sturzenegger hypothesis, how much dark

[5] See Hausmann Ricardo and Sturzenegger Federico, "U.S. and Global Imbalances: Can Dark Matter Prevent a Big Bang?," working paper CID (Center For International Development), Harvard University, 2005.

matter was there in 2010? Let $TNIIP$ denote the 'true' net international investment position and NIIP the recorded one. Then we have that

$$TNIIP = NIIP + \text{Dark Matter}$$

Let r denote the interest rate on net foreign assets. Then it must be true that

$$NII = r \times TNIIP$$

In this expression, we use TNIIP and not NIIP to calculate NII because, according to the dark-matter hypothesis, the recorded level of NII appropriately reflects the return on the true level of net international investment. In 2010, NII was 171.3 billion dollars (see table 1.1). Suppose that r is equal to 5 percent per year. Then, we have that $TNIIP = 171.3/0.05 = 3.4$ trillion dollars. Now the recorded NIIP in 2010 was -2.5 trillion dollars. This means that dark matter in 2010 was about 6 trillion dollars. This is a very big number to go under the radar of the Bureau of Economic Analysis!

1.5.2 Return Differentials

An alternative explanation for the paradoxical combination of positive NII and negative NIIP is that there is no dark matter, but that the United States earns a higher interest rate on its foreign asset holdings than foreigners earn on their U.S. asset holdings. The rationale behind this explanation is the observation that the U.S. international assets and liabilities are composed of different types of financial instruments. Specifically, the data show that foreign investors typically hold low-risk U.S. assets, such as Treasury Bills.

These assets carry a low interest rate. At the same time, American investors tend to purchase more risky foreign assets, such as foreign stocks, which earn relatively high returns.

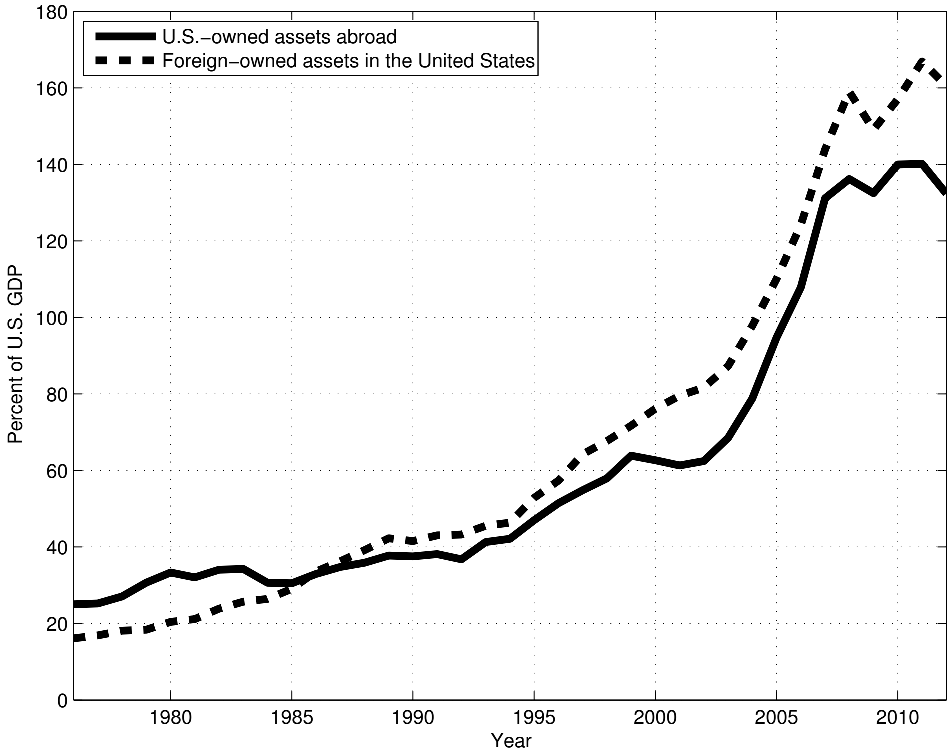
How big does the spread between the interest rate on U.S.-owned foreign assets and the interest rate on foreign-owned U.S. assets have to be to explain the paradox? Let A denote the U.S. gross foreign asset position and L the U.S. gross foreign liability position. Further, let r^A denote the interest rate on A and r^L the interest rate on L. Then, we have that

$$NII = r^A A - r^L L$$

How big does the spread $r^A - r^L$ have to be to explained the observed values of NII, A, and L with zero dark matter? We have data on the size of NII, A and L. Figure 1.6 shows the behavior of NII and figure 1.7 displays A_t/GDP_t and L_t/GDP_t in the United States for the period 1976 to 2012. The U.S. gross asset positions have grown very large since the 1990s from about 40 percent of GDP to more than 160 percent of GDP in the case of L_t and 140 percent of GDP in the case of A_t. Growth in the size of gross positions has been much larger than the growth in the net position. Recall that the net international investment position has fallen over this period from about -5 percent of GDP to -30 percent of GDP.[6] Suppose we set r^L equal to the return on one-year U.S. Treasury securities. For example,

[6]Massive growth in gross foreign asset and gross foreign liabilities is a recent phenomenon. As mentioned earlier several countries experienced large and persistent current account balances in the period 1870 to 1914. However, an important difference to the current period is that net foreign asset positions were very close to gross asset positions. (See, WEO, IMF, April 2005, page 119.)

Figure 1.7: U.S.-Owned Assets Abroad (A) and Foreign-Owned Assets in the U.S. (L)



Data Source: `http://www.bea.gov`.

in 2010, the U.S. gross foreign asset position (A) was 20.3 trillion dollars, whereas its gross foreign liability position (L) was 22.8 trillion dollars. The U.S. net investment income (NII) in that year was 191 billion and the rate on one-year Treasury securities was 0.32 percent. Then using the above expression, we have that r^A is the solution to

$$0.191 = r^A \times 20.3 - 0.0032 \times 22.8,$$

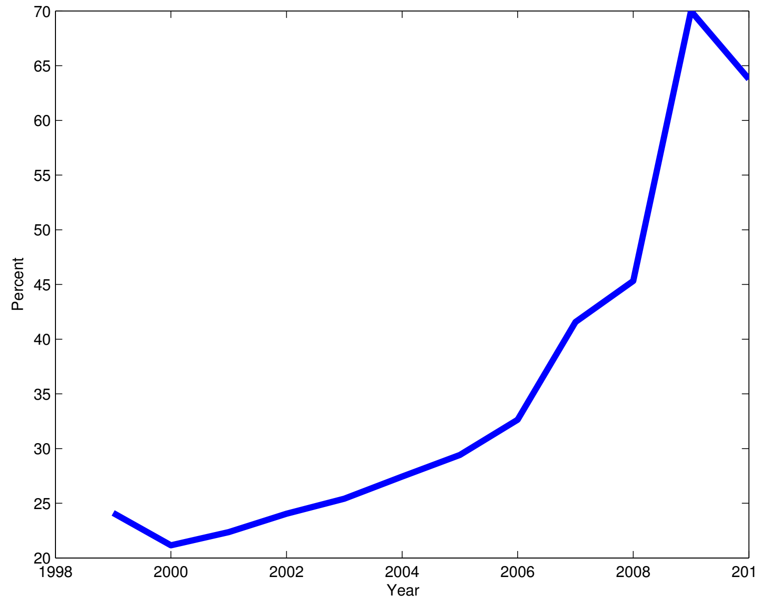
which yields $r^A = 1.3\%$. That is, we need an interest rate spread of 1 percentage point to explain the paradox. This figure seems more empirically plausible than 6 trillion dollars of dark matter.

1.6 Who Lends and Who Borrows Around the World?

The large observed U.S. current account deficits must be matched by current account surpluses of other countries with the United States. Over the past decade, an increasing fraction of the U.S. current account deficit is accounted for by current account deficits with China. Figure 1.8 displays the U.S. current account with China as a fraction of the total U.S. current account balance. This ratio was about 20 percent in 1999 and has been increasing steadily, reaching a peak of 70 percent in 2009.

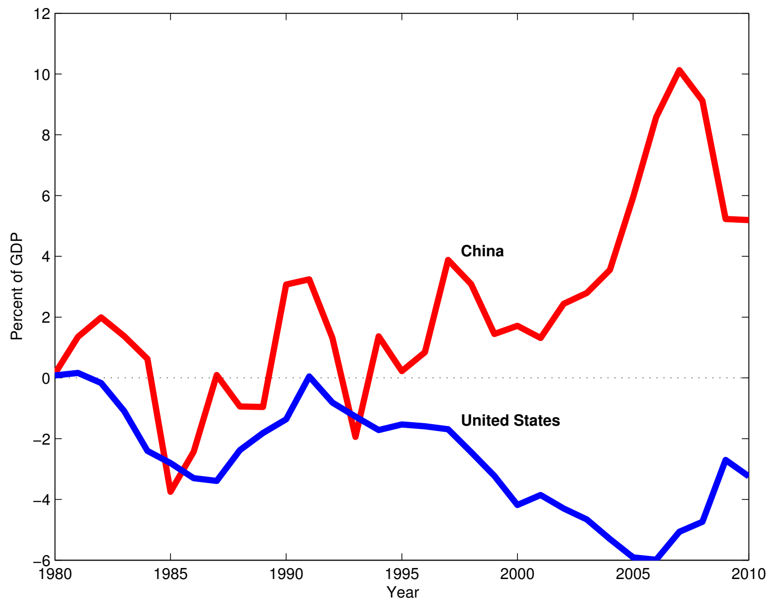
The expanding commercial relation between the United States and China has reached a magnitude such that the respective total current accounts are beginning to mirror each other. This phenomenon is evident from figure 1.9,

Figure 1.8: The U.S. Current Account Deficit With China



Source: `http://www.bea.gov`. Note: The U.S. current account deficit with China is expressed as a fraction of the total U.S. current account deficit.

Figure 1.9: The Current Accounts of China and the United States

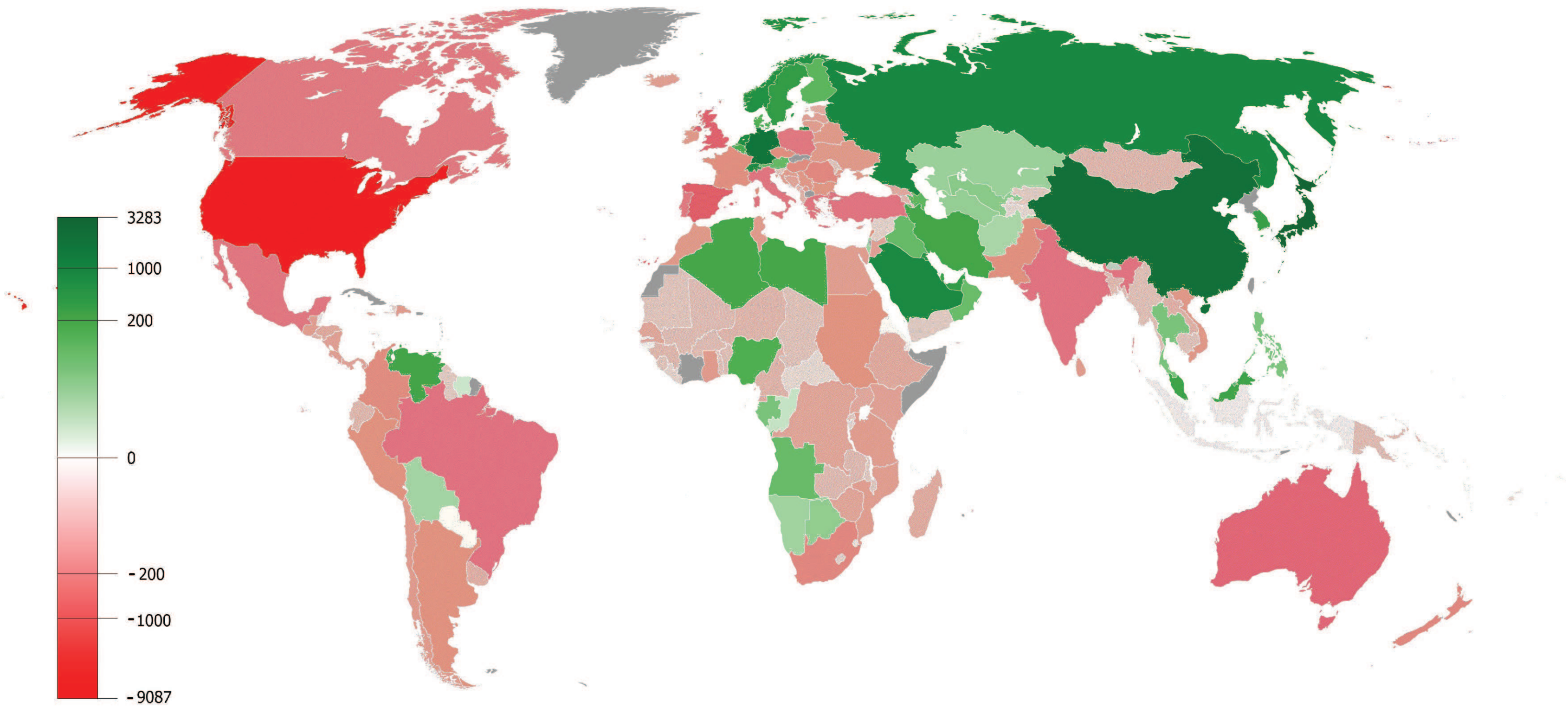


Source: `http://www.bea.gov`. Note: The current accounts of China and the United States are expressed as fractions of their respective GDPs.

which displays the current account balances of the United States and China as fractions of their respective GDPs. Since the mid 1990s, the U.S. widening current account deficits have coincided with a growing path of Chinese current account surpluses. Notice that the great recession of 2008-2009 was associated with a significant improvement in the U.S. current account and an equally important contraction in the Chinese current account surplus.

At a global level, all current account balances must add up to zero. It follows that by accumulating the current account balances of each country over time, we can obtain an idea of which countries have been playing the role of lenders and which the role of borrowers. The map in figure 1.10 presents this information. It shows the cumulative current account of each country in the world over the period 1980-2008. Cumulative surpluses appear in green and cumulative deficits in red. Darker tones correspond to larger cumulative deficits or surpluses. As expected, the U.S. appears in dark red and China in dark green. More generally, the pattern that emerges is that over the past three decades, the lenders of the world have been oil-exporting countries (Russia, the Middle East, some Scandinavian countries, and Venezuela), China, Japan, and Germany. The rest of the world has been borrowing from these countries. One way to interpret the map is that it demonstrates large global current account imbalances. If the long-run cumulative current account of most countries was in balance, then the map should be filled in with only light green and light red colors. The fact that the map has several very dark green and very dark red spots is therefore an indication of global current account imbalances. One may wonder how this map will look in the future. Will the debtor countries get out of the red, that is, will large current account deficits prove unsustainable? We take up this issue in the next chapter.

Figure 1.10: Cumulative Current Account Balances Around the World: 2008-2012



Note: The graph shows for each country the sum of current account balances in billions of U.S. dollars between 1980 and 2012. It was adapted from Source: `http://commons.wikimedia.org/wiki/File:Cumulative_Current_Account_Balance.png`) using the software `http://gunn.co.nz/map` .

1.7 Exercises

1. Describe how each of the following transactions affects the U.S. Balance of Payments. (Recall that each transaction gives rise to two entries in the Balance-of-Payments Accounts.)

   (a) An American university buys several park benches from Spain and pays with a \$120,000 check.

   (b) Floyd Townsend, of Tampa Florida, buys 5,000.00 dollars worth of British Airlines stock from Citibank New York, paying with U.S. dollars.

   (c) A French consumer imports American blue jeans and pays with a check drawn on a U.S. bank in New York.

   (d) An American company sells a subsidiary in the United States and with the proceeds buys a French company.

   (e) A group of American friends travels to Costa Rica and rents a vacation home for \$2,500. They pay with a U.S. credit card.

   (f) The United States sends medicine, blankets, tents, and nonperishable food worth 400 million dollars to victims of an earthquake in a foreign country.

   (g) Bonus items. The following two transactions involve a component of the balance of payments that we have ignored because it is quantitatively insignificant. If you decide to answer this question, take a look at footnote 1 in the book.

i. A billionaire from Russia enters the United States on an immigrant visa (that is, upon entering the United States she becomes a permanent resident of the United States.) Her wealth in Russia is estimated to be about 2 billion U.S. dollars.

ii. The United States forgives debt of $500,000 to Nicaragua.

2. In section 1.4, we showed that over the past 20 years the NIIP of the United States greatly benefited from valuation changes. In this question, you are asked to analyze how valuation changes affected the NIIP of China between 1981 and 2007. For the net foreign asset position of China use the time series constructed by Lane and Milesi-Ferretti (`http://www.philiplane.org/EWN.html`) rather than the China's official NIIP data. Current account data is available from the IMF's World Economic Outlook Database, which you should download. Use these two time series to construct a time series for valuation changes in China's net foreign asset position. Using a software package such as Excel or Matlab, plot a time series for the net foreign asset position, the cumulative current account, and valuation changes for China, since 1981. Make one graph in which the units are billions of U.S. dollars and one graph in which the units are percent of GDP. GDP data are also in the Lane and Milesi-Ferretti spreadsheet. Then use these graphs to contrasts the valuations changes experienced by China and by the United States. To which extend do your findings regarding the sign of the valuation changes support a view that on net China holds

more low risk/low return assets than high risk/high return assets.

3. **Dark Matter.** Following the same arguments as Hausmann and Sturzenegger, as presented in subsection 1.5.1, find the 'true' net international investment position of the United States for the years 2004 to 2010, assuming (like Hausmann and Sturzenegger) an annual interest rate of 5 percent. Annual data on net investment income for the period 2004 to 2010 can be found in lines 13 and 30 of Table 1. U.S. International Transactions (available online at `www.bea.gov`). Contrast the Hausmann and Sturzenegger implied net foreign asset position with that officially reported (see Table 2. International Investment Position of the United States at Year end,line1,available on courseworks or `www.bea.gov`.) In particular, discuss the year-to-year changes in the Hausmann-Sturzenegger measure of the net foreign asset position and contrast it to the current account balance and observed valuation changes.

4. This exercise asks you to look at the current account balances and the net foreign asset position of the GIPS (Greece, Ireland, Portugal, and Spain) countries. Your first data source is the IMF's World Economic Outlook Data Base (version April 2011). `http://www.imf.org`.

   (a) Obtain data for the current account to GDP ratio for each GIPS country for the period 1980 to 2010.

   (b) Use a computer software, such as Matlab or Excel, to plot the current account to GDP ratio against time for each country. Provide a short discussion of your graph.

(c) Find the cumulative CA balance for each of the four GIPS countries over the period 1980 to 2010 in U.S. dollars and discuss the implied change in the net foreign asset positions of each country and relate it to GDP in 2010.

The second data set contains information on the net foreign asset position (or net international investment position) of the GIPS countries. Specifically, use the data compiled by Philip L. Lane and Gian Maria Milesi Feretti, available online at `http://www.philiplane.org/EWN.html`.

(d) Again make a time series plot for each country of the NFA (net foreign asset position—as calculated by Milesi Feretti and Lane) in U.S. dollars.

(e) Compare the change in the NFA as calculated by Milesi Feretti and Lane with the CA balance data from the WEO database. What factors may explain these differences.

(f) Finally, assume it was the year 2005 and you looked at CA and NFA data for the GIPS countries. Now with the benefit of hindsight, what early signs of balance of payments trouble do you detect.

(g) Which of the GIPS countries were still running CA deficits after the onset of the financial crisis in 2009 and 2010. Who do you think is financing those current account deficits—the foreign private creditors or foreign governments?

Chapter 2

Current Account Sustainability

A natural question that arises from our description of the recent history of the U.S. external accounts is whether the observed trade balance and current account deficits are sustainable in the long run. In this chapter, we develop a simple framework to address this question.

2.1 Can a Country Run a Perpetual Trade Balance Deficit?

The answer to this question depends on the sign of a country's initial net international investment position. A negative net international investment position means that the country as a whole is a debtor to the rest of the world. Thus, the country must generate trade balance surpluses either currently or at some point in the future in order to service its foreign debt.

Similarly, a positive net international investment position means that the country is a net creditor of the rest of the world. The country can therefore afford to run trade balance deficits forever and finance them with the interest revenue generated by its credit position with the rest of the world.

Let's analyze this idea more formally. Consider an economy that lasts for only two periods, period 1 and period 2. Let TB_1 denote the trade balance in period 1, CA_1 the current account balance in period 1, and B_1^* the country's net international investment position (or net foreign asset position) at the end of period 1. For example, if the country in question was the United States and period 1 was meant to be 2012, then $CA_1 = -475$ billion, $TB_1 = -539.5$, and $B_1^* = -3,864$ billion (see table 1.1 and figure 1.3 in chapter 1). Let r denote the interest rate paid on investments held for one period and B_0^* denote the net foreign asset position at the end of period 0. Then, the country's net investment income in period 1 is given by

$$\text{Net investment income in period } 1 = rB_0^*.$$

This exresssion says that net investment income in period 1 is equal to the return on net foreign assets held by the country's residents between periods 0 and 1.

In what follows, we ignore net international payments to employees and net unilateral transfers by assuming that they are always equal to zero. Then, the current account equals the sum of net investment income and the trade balance, that is,

$$CA_1 = rB_0^* + TB_1. \tag{2.1}$$

The current account, in turn, represents the amount by which the country's net foreign asset position changes in period 1, that is,

$$CA_1 = B_1^* - B_0^*. \tag{2.2}$$

Here we are abstracting from valuation changes. Combining equations (2.1) and (2.2) to eliminate CA_1 yields:

$$B_1^* = (1+r)B_0^* + TB_1.$$

A relation similar to this one must also hold in period 2. So we have that

$$B_2^* = (1+r)B_1^* + TB_2.$$

Combining the last two equations to eliminate B_1^* we obtain

$$(1+r)B_0^* = \frac{B_2^*}{(1+r)} - TB_1 - \frac{TB_2}{(1+r)}. \tag{2.3}$$

Now consider the possible values that the net foreign asset position at the end of period 2, B_2^*, can take. If B_2^* is negative ($B_2^* < 0$), it means that in period 2 the country is acquiring debt to be paid off in period 3. However, in period 3 nobody will be around to collect the debt because the world ends in period 2. Thus, the rest of the world will not be willing to lend to our country in period 2. This means that B_2^* cannot be negative, or that B_2^* must satisfy $B_2^* \geq 0$. This restriction is known as the no-Ponzi-game condition.[1] Can B_2^* be strictly positive? The answer is no. A positive value of B_2^* means that the country is lending to the rest of the world in period 2. But clearly the country will be unable to collect this debt in period 3 because, again, the world ends in period 2. Thus, the country will never choose to hold a positive net foreign asset position at the end of period 2, that is, it must be the case that $B_2^* \leq 0$. If B_2^* can be neither positive nor negative, it must be equal to zero:

$$B_2^* = 0.$$

This condition is known as the transversality condition. Using this expression, (2.3) becomes

$$(1+r)B_0^* = -TB_1 - \frac{TB_2}{(1+r)}. \tag{2.4}$$

This equation states that a country's initial net foreign asset position must equal the present discounted value of its future trade deficits. Our claim that a negative initial net foreign wealth position implies that the country must generate trade balance surpluses, either currently or at some point in the future, can be easily verified using equation (2.4). Suppose that the

[1]This constraint on terminal asset holdings is named after Charles K. Ponzi, who introduced pyramid schemes in the 1920s in Massachusetts. To learn more about the remarkable criminal career of Ponzi, visit `http://www.mark-knutson.com`. A recent example of a Ponzi scheme is given by Bernard L. Madoff's fraudulent squandering of investments valued around \$64 billion in 2008. For more than 20 years Madoff's scheme consisted in paying steady returns slightly above market to a large variety of clients ranging from hedge funds to university endowments to low-income retirees. When Madoff's own investments failed to produce such returns, the scheme required the acquisition of new clients to survive. In the financial crisis of 2008 the flow of new clients dried up and his scheme imploded overnight. In June 2009, Bernard Madoff, then 71 years old, was sentenced to 150 years in prison.

country is a net debtor to the rest of the world ($B_0^* < 0$). Clearly, if it never runs a trade balance surplus ($TB_1 \leq 0$ and $TB_2 \leq 0$), then the left-hand side of (2.4) is negative while the right-hand side is positive, so (2.4) would be violated. In this case, the country would be running a Ponzi scheme against the rest of the world.

Now suppose that the country's initial asset position is positive ($B_0^* > 0$). This means that initially the rest of the world owes a debt to our country. Then, the left-hand side of equation (2.4) is positive. If the country runs trade deficits in periods 1 and 2, then the right hand side of (2.4) is also positive, which implies no inconsistency. Thus, the answer to the question of whether a country can run a perpetual trade balance deficit is yes, provided the country's initial net foreign asset position is positive. Because the U.S. is currently a net foreign debtor to the rest of the world, it follows that it will have to run trade balance surpluses at some point in the future. This result extends to economies that last for any number of periods, not just two. Indeed, the appendix to this chapter shows that the result holds for economies that last forever (infinite-horizon economies).

2.2 Can a Country Run a Perpetual Current Account Deficit?

In a finite-horizon economy like the two-period world we are studying here, the answer to this question is, again, yes, provided the country's initial net foreign asset position is positive. To see why, note that an expression similar to (2.2) must also hold in period 2, that is,

$$CA_2 = B_2^* - B_1^*.$$

Combining this expression with equation (2.2) to eliminate B_1^*, we obtain

$$B_0^* = -CA_1 - CA_2 + B_2^*.$$

Imposing the transversality condition, $B_2^* = 0$, it follows that

$$B_0^* = -CA_1 - CA_2. \tag{2.5}$$

This equation says that a country's initial net foreign asset position must be equal to the sum of its current account deficits. Suppose the country's initial net foreign asset position is negative, that is, $B_0^* < 0$. Then for this country to satisfy equation (2.5) the sum of its current account surpluses must be positive ($CA_1 + CA_2 > 0$), that is, the country must run a current account surplus in at least one period. However, if the country's initial asset position is positive, that is, if $B_0^* > 0$, then the country can run a current account deficit in both periods (which in the present two-period economy is tantamount to perpetually).

This result is valid for any finite horizon. However, the appendix shows that in an infinite horizon economy, a negative initial net foreign asset position does not preclude an economy from running perpetual current account deficits. What is needed for the country not to engage in a Ponzi scheme is that it pay periodically part of the interest accrued on its net foreign debt to ensure that the foreign debt grows at a rate less than the interest rate. In this way, the present discounted value of the country's debt would be zero, which is to say that in present-discounted-value terms the country would pay its debt. Because in this situation the country's net foreign debt is growing over time, the economy must devote an ever larger amount of resources (i.e., it must generate larger and larger trade surpluses) to servicing part of its interest obligations with the rest of the world. The need to run increasing trade surpluses over time requires domestic output to also grow over time. For if output did not grow, the required trade balance surpluses would eventually exceed GDP, which is impossible.

2.3 Savings, Investment, and the Current Account

In this section, we show how to link, using accounting identities, the current account to a number of familiar macroeconomic aggregates, such as savings, investment, gross domestic product (GDP) and domestic absorption. These accounting identities allow us to view current account deficits from a number of perspectives and will be of use when studying the determination of the current account in a general equilibrium model.

2.3.1 Current Account Deficits As Declines in the Net International Investment Position

Recall the basic concept, introduced earlier, that in the absence of valuation changes, the current account measures the change in the net international investment position of a country:

$$CA_t = B_t^* - B_{t-1}^*,$$

where CA_t denotes the country's current account in period t and B_t^* the country's net international investment position at the end of period t. If the current account is in deficit, $CA_t < 0$, then the net international investment position falls, $B_t^* - B_{t-1}^* < 0$. Similarly, if the current account displays a surplus, $CA_t > 0$, then the net international investment position improves, $B_t^* - B_{t-1}^* > 0$.

2.3.2 Current Account Deficits As Reflections of Trade Deficits

All other things equal, larger trade imbalances, or a larger gap between imports and exports, are reflected in larger current account deficits. This follows from the definition of the current account. The current account is equal to the sum of the trade balance and net investment income (again, we are ignoring net international compensation to employees and net unilateral transfers):

$$CA_t = TB_t + rB_{t-1}^*, \tag{2.6}$$

where TB_t denotes the trade balance in period t, and r denotes the interest rate. Figure 1.1 from chapter 1 shows that in the United States, the trade balance and the current account move closely together.

2.3.3 The Current Account As The Gap Between Savings and Investment

The current account is in deficit when investment exceeds savings. To see this, begin by recalling from chapter 1 that the trade balance equals the difference between exports and imports of goods and services. Letting X_t denote exports in period t and IM_t denote imports in period t, we then have that

$$TB_t = X_t - IM_t.$$

Let Q_t denote the amount of final goods and services produced domestically in period t. This measure of output is typically referred to as gross domestic product, or GDP. Let C_t denote the amount of goods and services consumed domestically by the private sector in period t, G_t denote government consumption in period t, and I_t denote the amount of goods and services used for domestic investment (in plants, infrastructure, etc.) in period t. We will refer to C_t, G_t, and I_t simply as consumption, government spending, and investment in period t, respectively. Then we have that

$$Q_t + IM_t = C_t + I_t + G_t + X_t.$$

This familiar identity, states that the aggregate supply of goods, given by the sum of GDP and imports, can be used in four ways, private consumption, investment, public consumption, or exports. Using the fact that the $TB_t =$

$X_t - IM_t$ and rearranging, we obtain

$$TB_t = Q_t - C_t - I_t - G_t. \tag{2.7}$$

Plugging this relation into equation (2.6) yields

$$CA_t = rB^*_{t-1} + Q_t - C_t - I_t - G_t.$$

The sum of GDP and net investment income (rB^*_{t-1}), is called national income, or gross national product (GNP). We will denote national income in period t by Y_t, that is,

$$Y_t = Q_t + rB^*_{t-1}.$$

Combining the last two expressions results in the following representation of the current account

$$CA_t = Y_t - C_t - I_t - G_t. \tag{2.8}$$

National savings, which we will denote by S_t, is defined as the difference between national income and the sum of private and government consumption, that is,

$$S_t = Y_t - C_t - G_t.$$

It then follows from equation (2.8) that the current account is equal to savings minus investment,

$$CA_t = S_t - I_t. \tag{2.9}$$

According to this relation, a deficit in the current account occurs when a country's investment exceeds its savings. Conversely, a current account surplus obtains when a country's investment falls short of its savings.

2.3.4 The Current Account As the Gap Between National Income and Domestic Absorption

The current account is in deficit when domestic absorption of goods and services exceeds national income. A country's absorption, which we denote by A_t, is defined as the sum of private consumption, government consumption, and investment,

$$A_t = C_t + I_t + G_t.$$

Combining this definition with equation (2.8), the current account can be expressed as the difference between income and absorption:

$$CA_t = Y_t - A_t. \tag{2.10}$$

2.3.5 Four Ways of Viewing the Current Account

Summing up, we have derived four alternative expressions for the current account:

$$
\begin{aligned}
CA_t &= B_t^* - B_{t-1}^* \\
CA_t &= rB_{t-1}^* + TB_t \\
CA_t &= S_t - I_t \\
CA_t &= Y_t - A_t
\end{aligned}
$$

which emphasize the relationship between the current account and alternative macroeconomic aggregates: respectively, the accumulation of foreign assets, the trade balance, savings and investment, and income and absorption. All four of the above expressions represent accounting identities that must be satisfied at all times in any economy. They do not provide any explanation, or theory, of the determinants of the current account. To explain the behavior of the current account we need a model, that is, a story of the economic behavior of households, firms, governments, and foreign residents. This is the focus of the following chapters.

2.4 Appendix: Perpetual Trade-Balance and Current-Account Deficits in Infinite-Horizon Economies

In a world that lasts for only 2 periods, forever means for periods 1 and 2. Therefore, in such a world a country runs a perpetual trade deficit if the trade balance is negative in periods 1 and 2. Similarly, in a two-period world a country runs a perpetual current account deficit if it experiences a negative current account balance in periods 1 and 2. In the body of this chapter, we showed that a two-period economy can run a perpetual trade balance deficit only if it starts with a positive net international investment position. A similar condition holds for the current account: a two-period country can run a perpetual current account deficit only if its initial net international asset position is positive. In this appendix we study how these results change in a more realistic setting in which the economy lasts for an infinite number of periods.

Suppose that the economy starts in period 1 and lasts indefinitely. The net foreign asset position at the end of period 1 takes the familiar form

$$B_1^* = (1+r)B_0^* + TB_1,$$

Solve for B_0^* to obtain

$$B_0^* = \frac{B_1^*}{1+r} - \frac{TB_1}{1+r}. \tag{2.11}$$

Now shift this expression one period forward to obtain

$$B_1^* = \frac{B_2^*}{1+r} - \frac{TB_2}{1+r}.$$

Use this formula to eliminate B_1^* from equation (2.11) to obtain

$$B_0^* = \frac{B_2^*}{(1+r)^2} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2}.$$

Shifting (2.11) two periods forward yields

$$B_2^* = \frac{B_3^*}{1+r} - \frac{TB_3}{1+r}.$$

Combining this expression with the one right above it, we obtain

$$B_0^* = \frac{B_3^*}{(1+r)^3} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \frac{TB_3}{(1+r)^3}$$

Repeating this iterative procedure T times results in the relationship

$$B_0^* = \frac{B_T^*}{(1+r)^T} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \cdots - \frac{TB_T}{(1+r)^T}. \tag{2.12}$$

In an infinite-horizon economy, the no-Ponzi-game constraint becomes

$$\lim_{T\to\infty} \frac{B_T}{(1+r)^T} \geq 0.$$

This expression says that the net foreign debt of a country must grow at a rate less than r. Note that having a debt that grows at the rate r (or higher) is indeed a scheme in which the principal and the interest accrued on the debt are perpetually rolled over. That is, it is a scheme whereby the debt is never paid off. The no-Ponzi-game constraint precludes this type of situations.

At the same time, the country will not want to have a net credit with the rest of the world growing at a rate r or higher, because that would mean that the rest of the world forever rolls over its debt with the country in question. This means that the path of net investment positions must satisfy

$$\lim_{T\to\infty} \frac{B_T}{(1+r)^T} \leq 0.$$

This restriction and the no-Ponzi-game constraint can be simultaneously satisfied only if the following transversality condition holds:

$$\lim_{T\to\infty} \frac{B_T}{(1+r)^T} = 0.$$

Letting T go to infinity and using this transversality condition, equation (2.12) becomes

$$B_0^* = -\frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \dots$$

This expression states that the initial net foreign asset position of a country must equal the present discounted value of the stream of current and future expected trade deficits. Clearly, if the initial foreign asset position of the country is negative ($B_0^* < 0$), then the country must run trade balance surpluses at some point. We conclude that regardless of whether we consider a finite horizon economy or an infinite horizon economy, a country that starts with a negative net foreign asset position cannot run perpetual trade balance deficits.

We next revisit the question of whether a country can run perpetual current account deficits. We can write the evolution of the country's net foreign asset position in a generic period $t = 1, 2, 3, \ldots$ as

$$B_t^* = (1 + r)B_{t-1}^* + TB_t.$$

Suppose that the initial net foreign asset position of the country, B_0^*, is negative. That is, the country starts out as a net debtor to the rest of the world. Consider an example in which each period the country generates a trade balance surplus sufficient to pay a fraction α of its interest obligations. That is,

$$TB_t = -\alpha r B_{t-1}^*,$$

where the factor α is between 0 and 1. Note that according to this expression, whenever the country is a net debtor to the rest of the world, i.e., whenever $B_{t-1}^* < 0$, it generates a trade balance surplus. Combining this policy with the evolution of the net asset position, we obtain

$$B_t^* = (1 + r - \alpha r)B_{t-1}^*.$$

Because B_0^* is negative and because $1+r-\alpha r$ is positive, we have that the net foreign asset position of the country will be forever negative. Furthermore, each period the country runs a current account deficit. To see this, recall that the current account is given by $CA_t = rB_{t-1}^* + TB_t$, which, given the assumed debt-servicing policy, results in

$$CA_t = r(1 - \alpha)B_{t-1}^* < 0.$$

A natural question is whether the country is satisfying the transversality condition. The law of motion of B_t^* given above implies that

$$B_t^* = (1 + r - \alpha r)^t B_0^*.$$

It follows that

$$\frac{B_t^*}{(1+r)^t} = \left[\frac{1 + r(1-\alpha)}{1+r}\right]^t B_0^*,$$

which converges to zero as t becomes large because $1 + r > 1 + r(1 - \alpha)$.

Notice that under the assumed policy the trade balance evolves according to

$$TB_t = -\alpha r[1 + r(1 - \alpha)]^{t-1} B_0^*.$$

That is, the trade balance is positive and grows unboundedly over time at the rate $r(1 - \alpha)$. In order for a country to be able to generate this path of trade balance surpluses, its GDP must be growing over time at a rate equal or greater than $r(1 - \alpha)$. If this condition is satisfied, the repayment policy described in this example would support perpetual current account deficits even if the initial net foreign asset position is negative.

2.5 Exercises

1. Consider a two-period economy that has at the beginning of period 1 a net foreign asset position of -100. In period 1, the country runs a current account deficit of 5 percent of GDP, and GDP in both periods is 120. Assume the interest rate in periods 1 and 2 is 10 percent.

   (a) Find the trade balance in period 1 (TB_1), the current account balance in period 1 (CA_1), and the country's net foreign asset position at the beginning of period 2 (B_1^*).

   (b) Is the country living beyond its means? To answer this question find the country's current account balance in period 2 and the associated trade balance in period 2. Is this value for the trade balance feasible? [Hint: Keep in mind that the trade balance cannot exceed GDP.]

   (c) Now assume that in period 1, the country runs instead a much larger current account deficit of 10 percent of GDP. Find the country's net foreign asset position at the end of period 1, B_1^*. Is the country living beyond its means? If so, show why.

Chapter 3

A Theory of Current Account Determination

In this chapter, we build a model of an open economy to study the determinants of the trade balance and the current account. In particular, we study the response of the trade balance and the current account to a variety of economic shocks, such as changes in income and the world interest rate. We pay special attention to how those responses depend on whether the shocks are temporary or permanent.

3.1 A Small Open Economy

We say that an economy is open when it trades in goods and financial assets with the rest of the world. We say that an economy is small when world prices and interest rates are independent of the level of any domestic economic variable. Most countries in the world are small open economies.

Examples of highly developed small open economies are the Netherlands, Switzerland, Austria, New Zealand, Australia, Canada, and Norway. Examples of emerging small open economies are Chile, Peru, Bolivia, Greece, Portugal, Estonia, Latvia, and Thailand. Large open economies are the United States, Japan, Germany, and the United Kingdom. China is an example of a large economy that is in transition from closed to open. There are not many examples of completely closed economies. Perhaps the most notable cases are North Korea, Cuba, and Iran. The economic size of an economy may not be related to its geographic size. For example, Australia and Canada are geographically large, but economically small. On the other hand, Japan, Germany, the United Kingdom, and France, are geographically small, but economically large. Also, demographic and economic size may not be correlated. For example, India is demographically large, but remains economically small.

Consider a small open economy in which people live for two periods, 1 and 2, and are endowed with Q_1 units of goods in period 1 and Q_2 units in period 2. Goods are assumed to be perishable in the sense that they cannot be stored from one period to the next. In addition, households are assumed to be endowed with B_0^* units of a bond. In period 1, these bond holdings generate interest income in the amount of $r_0 B_0^*$, where r_0 denotes the interest rate on bonds held between periods 0 and 1. In period 1, the household's income is given by the sum of interest on its bond holdings and its endowment of goods, $r_0 B_0^* + Q_1$. The household can allocate its income to two alternative uses: purchases of consumption goods, which we denote by C_1, and purchases of bonds, $B_1^* - B_0^*$, where B_1^* denotes bond holdings at the end of period 1. Thus, in period 1 the household faces the following budget constraint:

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + Q_1. \tag{3.1}$$

Similarly, in period 2 the representative household faces a constraint stating that consumption expenditure plus bond purchases must equal income:

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + Q_2, \tag{3.2}$$

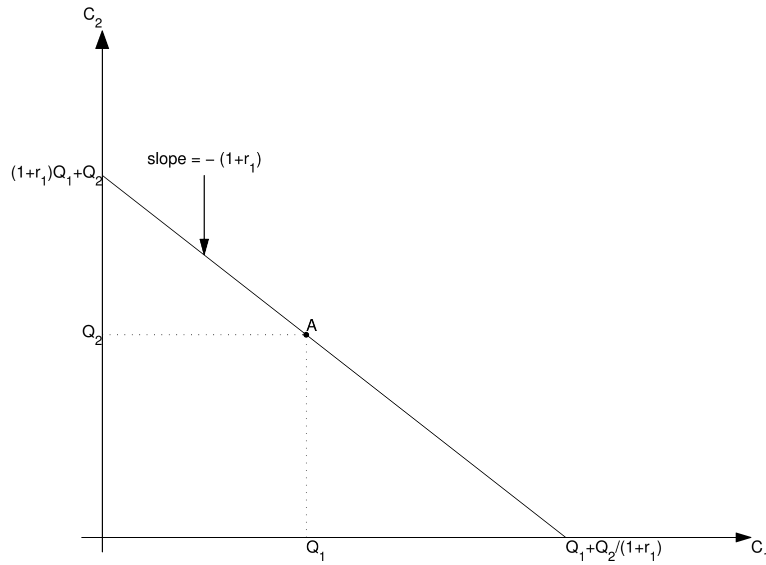
where C_2 denotes consumption in period 2, r_1 denotes the interest rate on assets held between periods 1 and 2, and B_2^* denotes bond holdings at the end of period 2. As explained in chapter 1, by the no-Ponzi-game constraint households are not allowed to leave any debt at the end of period 2, that is, B_2^* must be greater than or equal to zero. Also, because the world is assumed to last for only 2 periods, agents will choose not to hold any positive amount assets at the end of period 2, as they will not be around in period 3 to spend those savings in consumption. Thus, asset holdings at the end of period 2 must be exactly equal to 0:

$$B_2^* = 0. \tag{3.3}$$

Combining the budget constraints (3.1) and (3.2) and the terminal condition (3.3) to eliminate B_1^* and B_2^*, gives rise to the following lifetime budget constraint of the household:

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}. \tag{3.4}$$

Figure 3.1: The intertemporal budget constraint

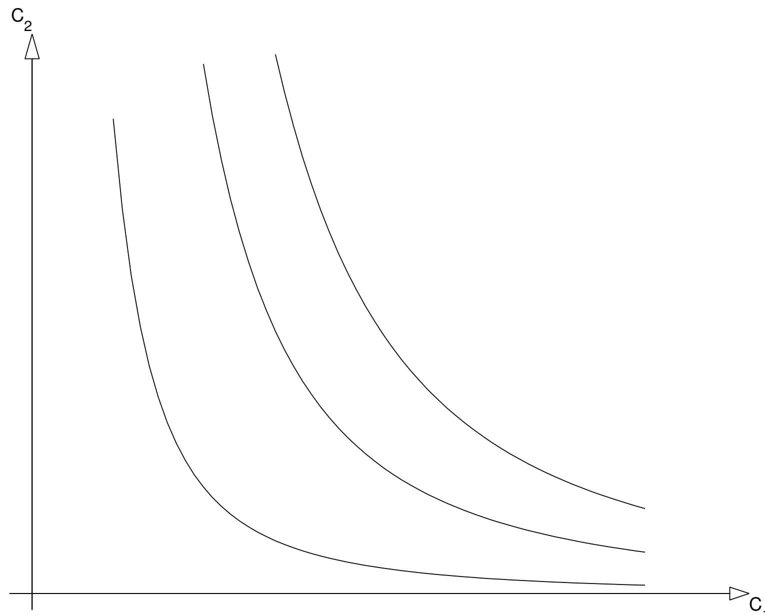


This intertemporal budget constraint requires that the present discounted value of consumption (the left-hand side) be equal to the initial stock of wealth plus the present discounted value of the endowment stream (the right-hand side). The household chooses consumption in periods 1 and 2, C_1 and C_2, taking as given all other variables appearing in (3.4), namely, r_0, r_1, B_0^*, Q_1, and Q_2.

Figure 3.1 displays the pairs (C_1, C_2) that satisfy the household's intertemporal budget constraint (3.4). For simplicity, we assume for the remainder of this section that the household's initial asset position is zero, that is, we assume that $B_0^* = 0$. Then, clearly, the basket $C_1 = Q_1$ and $C_2 = Q_2$ (point A in the figure) is feasible in the sense that it satisfies the intertemporal budget constraint (3.4). In words, the household can eat his endowment in each period. But the household's choices are not limited to this particular basket. In period 1 the household can consume more or less than Q_1 by borrowing or saving the amount $C_1 - Q_1$. If the household wants to increase consumption in one period, it must sacrifice some consumption in the other period. In particular, for each additional unit of consumption in period 1, the household has to give up $1+r_1$ units of consumption in period 2. This means that the slope of the budget constraint is $-(1+r_1)$. Note that points on the budget constraint located southeast of point A correspond to borrowing (or dissaving) in period 1. Letting S_1 denote savings in period 1, we have that $S_1 = r_0 B_0^* + Q_1 - C_1 = Q_1 - C_1 < 0$ (recall that we are assuming that $B_0^* = 0$). At the same time, the fact that $S_1 < 0$ implies, by the relation $S_1 = B_1^* - B_0^*$, that the household's asset position at the end of period 1, B_1^*, is negative. This in turn implies that a point on the budget constraint located southeast of the endowment point A is also associated with positive saving in period 2 because $S_2 = B_2^* - B_1^* = -B_1^* > 0$ (recall that $B_2^* = 0$). On the other hand, points on the budget constraint located northwest of A are associated with positive saving in period 1 and dissaving in period 2. If the household chooses to allocate its entire lifetime income to consumption in period 1, then C_1 would equal $Q_1 + Q_2/(1+r_1)$ and C_2 would be nil. This point corresponds to the intersection of the budget constraint with the horizontal axis. If the household chooses to allocate all its lifetime income to consumption in period 2, then C_2 would equal $(1 + r_1)Q_1 + Q_2$ and C_1 would be nil. This basket is located at the intersection of the budget constraint with the vertical axis.

Which consumption bundle on the budget constraint the household will choose depends on its preferences. We will assume that households like both

Figure 3.2: Indifference curves

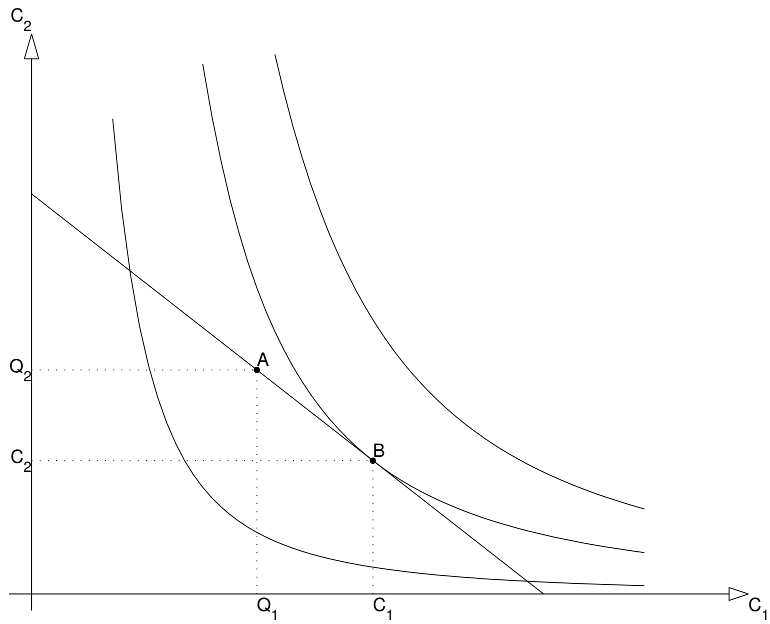


C_1 and C_2 and that their preferences can be described by the utility function

$$U(C_1, C_2), \tag{3.5}$$

where the function U is strictly increasing in both arguments. Figure 3.2 displays the household's indifference curves. All consumption baskets on a given indifference curve provide the same level of utility. Because consumption in both periods are goods, that is, items for which more is preferred to less, as one moves northeast in figure 3.2, utility increases. Note that the indifference curves drawn in figure 3.2 are convex toward the origin, so that at low levels of C_1 relative to C_2 the indifference curves are steeper than at relatively high levels of C_1. Intuitively, the convexity of the indif-

Figure 3.3: Equilibrium in the endowment economy



ference curves means that at low levels of consumption in period 1 relative to consumption in period 2, the household is willing to give up relatively many units of period-2 consumption for an additional unit of period-1 consumption. On the other hand, if period-1 consumption is high relative to period-2 consumption, then the household will not be willing to sacrifice much period-2 consumption for an additional unit of period-1 consumption. The negative of the slope of an indifference curve is known as the marginal rate of substitution of C_2 for C_1. Therefore, the assumption of convexity means that along a given indifference curve, the marginal rate of substitution decreases with C_1.

Households choose C_1 and C_2 so as to maximize the utility function (3.5) subject to the lifetime budget constraint (3.4). Figure 3.3 displays the lifetime budget constraint together with the household's indifference curves. At the feasible basket that maximizes the household's utility, the indifference curve is tangent to the budget constraint (point B).

Formally, the tangency between the budget constraint and the indifference curve is given by the following first-order condition of the household's maximization problem:

$$U_1(C_1, C_2) = (1 + r_1)U_2(C_1, C_2), \tag{3.6}$$

where $U_1(C_1, C_2)$ and $U_2(C_1, C_2)$ denote the marginal utilities of consumption in periods 1 and 2, respectively. The marginal utility of consumption in period 1 indicates the increase in utility resulting from the consumption of an additional unit of C_1 holding constant C_2. Similarly, the marginal utility of period 2 consumption represents the increase in utility associated with a unit increase in C_2 holding constant C_1. Technically, the marginal utilities of C_1 and C_2 are defined as the partial derivatives of $U(C_1, C_2)$ with respect to C_1 and C_2, respectively. That is,

$$U_1(C_1, C_2) = \frac{\partial U(C_1, C_2)}{\partial C_1}$$

and

$$U_2(C_1, C_2) = \frac{\partial U(C_1, C_2)}{\partial C_2}.$$

The ratio $\frac{U_1(C_1,C_2)}{U_2(C_1,C_2)}$ represents the negative of the slope of the indifference curve at the basket (C_1, C_2), or the marginal rate of substitution of C_2 for C_1. To see that (3.6) states that at the optimum the indifference curve is tangent to the budget constraint, divide the left and right hand sides of that equation by $-U_2(C_1, C_2)$ to obtain

$$-\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = -(1 + r_1)$$

and recall that $-(1 + r_1)$ is the slope of the budget constraint.

Condition (3.6) is quite intuitive. Suppose that the consumer sacrifices one unit of consumption in period 1 and saves it by buying a bond paying the interest rate r_1 in period 2. Then his utility in period 1 falls by $U_1(C_1, C_2)$. In period 2, he receives $1 + r_1$ units of consumption each of which gives him $U_2(C_1, C_2)$ units of utility, so that his utility in period 2 increases by $(1 + r_1)U_2(C_1, C_2)$. If the left-hand side of (3.6) is greater than the right-hand side, then the consumer can increase his lifetime utility by saving less (and hence consuming more) in period 1. Conversely, if the left-hand side of (3.6) is less than the right-hand side, then the consumer will be better off saving more (and consuming less) in period 1. At the optimal allocation, the left- and right-hand sides of (3.6) must be equal to each other, so that in the margin the consumer is indifferent between consuming an extra unit in period 1 and consuming $1 + r_1$ extra units in period 2.[1]

[1] One way of obtaining (3.6) is to solve for C_2 in (3.4) and to plug the result in the utility function (3.5) to get rid of C_2. The resulting expression is $U(C_1, (1+r_0)(1+r_1)B_0^* + (1 + r_1)Q_1 + Q_2 - (1 + r_1)C_1)$ and depends only on C_1 and other parameters that the household takes as given. Taking the derivative of this expression with respect to C_1 and setting it equal to zero—which is a necessary condition for a maximum—yields (3.6).

3.1.1 Equilibrium

We assume that all households are identical. Thus, by studying the behavior of an individual household, we are also learning about the behavior of the country as a whole. For this reason, we will not distinguish between the behavior of an individual household and that of the country as a whole. To keep things simple, we further assume that there is no investment in physical capital.[2]

We assume that the country has free access to international financial markets. This means that in equilibrium the domestic interest rate, r_1, must be equal to the world interest rate, which we will denote by r^*, that is,

$$r_1 = r^*.$$

If this condition is satisfied we will say that *interest rate parity* holds. The country is assumed to be sufficiently small so that its savings decisions do not affect the world interest rate. Because all households are identical, at any point in time all domestic residents will make identical saving decisions. This implies that domestic households will never borrow or lend from one another and that all borrowing or lending takes the form of purchases or sales of foreign assets. Thus, we can interpret B_t^* $(t = 0, 1, 2)$ as the country's net foreign asset position (or, in the terminology of the Bureau of Economic Analysis, the country's net international investment position), at the end of period t. Furthermore, the assumption that all households are identical implies that the intertemporal budget constraint of an individual household,

[2]In chapter 5, we will allow for production and capital accumulation.

given by equation (3.4), can be interpreted as the country's intertemporal resource constraint.

An equilibrium then is a consumption bundle (C_1, C_2) and an interest rate r_1 that satisfy the country's intertemporal resource constraint, the household's first-order condition for utility maximization, and interest rate parity, that is,

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1},$$

$$U_1(C_1, C_2) = (1+r_1)U_2(C_1, C_2),$$

and

$$r_1 = r^*,$$

given the exogenous variables $\{r_0, B_0^*, Q_1, Q_2, r^*\}$. Here, the term *exogenous* refers to variables whose values are determined outside of the model. For instance, the initial net foreign asset position B_0^*, is determined in period 0, before the consumers in our economy were born. The world interest rate, r^*, is determined in world financial markets, which our economy cannot affect because it is too small. And the endowment levels, Q_1 and Q_2 represent manna-type receipts of goods whose quantity and timing lies outside of consumers' control.

It is useful at this point to revisit the basic balance-of-payments accounting in our two-period model. We first show that the intertemporal resource constraint of the country can be expressed in terms of current and expected future trade balances. Begin by rearranging terms in the intertemporal resource constraint (3.4) to express it in the form

$$(1 + r_0)B_0^* = -(Q_1 - C_1) - \frac{(Q_2 - C_2)}{1 + r_1}.$$

In our simple economy, the trade balance in period 1 equals the difference between the endowment of goods in period 1, Q_1, and consumption of goods in period 1, C_1, that is, $TB_1 = Q_1 - C_1$. Similarly, the trade balance in period 2 is given by $TB_2 = Q_2 - C_2$. Using these expressions for TB_1 and TB_2 and recalling that in equilibrium $r_1 = r^*$, we can write the country's intertemporal resource constraint as:

$$(1 + r_0)B_0^* = -TB_1 - \frac{TB_2}{1 + r^*}. \tag{3.7}$$

This expression, which should be familiar from chapter 2, states that a country's present discounted value of trade deficits (the right-hand side) must equal its initial net foreign asset position including net investment income (the left-hand side). If the country starts out as a debtor of the rest of the world ($B_0^* < 0$), then it must run a trade surplus in at least one period in order to repay its debt ($TB_1 > 0$ or $TB_2 > 0$ or both). Conversely, if at the beginning of period 1 the country is a net creditor ($B_0^* > 0$), then it can use its initial wealth to finance current or future trade deficits. In particular, it need not run a trade surplus in either period. In the special case in which the country starts with a zero stock of foreign wealth ($B_0^* = 0$), a trade deficit in one period must be offset by a trade surplus in the other period.

The country's intertemporal resource constraint can also be written in terms of the current account. To do this, recall that the current account is equal to the sum of net investment income and the trade balance. Thus in period 1 the current account is given by $CA_1 = r_0 B_0^* + TB_1$ and the current account in period 2 is given by $CA_2 = r^* B_1^* + TB_2$. Using these two definitions to eliminate TB_1 and TB_2 from equation (3.7) yields

$$(1 + r_0)B_0^* = -(CA_1 - r_0 B_0^*) - \frac{(CA_2 - r^* B_1^*)}{1 + r^*}.$$

Note that the term $r_0 B_0^*$ appears on the left- and right-hand sides, and can therefore be eliminated. Also, using the definition $CA_2 = B_2^* - B_1^*$ to eliminate B_1^* and recalling that in equilibrium $B_2^* = 0$, we obtain, after collecting terms,

$$B_0^* = -CA_1 - CA_2.$$

This alternative way of writing the intertemporal resource constraint makes it clear that if the country is an initial debtor, then it must run a current account surplus in at least one period ($CA_1 > 0$ or $CA_2 > 0$). On the other hand, if the country starts out as a net creditor to the rest of the world, then it can run current and/or future current account deficits. Finally, if the country begins with no foreign debt or assets ($B_0^* = 0$), a current account deficit in one period must be offset by a current account surplus in the other period.

The equilibrium in our small open economy is shown as point B in figure 3.3. At the equilibrium allocation, the country runs a trade deficit in period 1, that is, $Q_1 - C_1$ is negative. Also, recalling our maintained assumption that foreign asset holdings in period 0 are nil ($B_0^* = 0$), the current account in period 1 equals the trade balance in that period ($CA_1 = r_0 B_0^* + TB_1 = TB_1$). Thus, in equilibrium, the current account is in deficit in period 1. In turn, the current account deficit in period 1 implies that the country starts period 2 as a net debtor to the rest of the world. As a result, in period 2 the country must generate a trade surplus to repay the debt plus interest, that is, $TB_2 = Q_2 - C_2 > 0$.

3.2 Temporary Versus Permanent Output Shocks

What is the effect on the current account of an increase in output? It turns out that this question, as formulated, is incomplete, and, as a result, does not have a clear answer. The reason is that in a world in which agents make decisions based on current and future expected changes in the economic environment, one needs to specify not only what the current change in the environment is, but also what the future expected changes are. The information that current output changes does not tell us in what direction, if any, future output is expected to move. Consider the following example. The income earner of a family falls ill and therefore cuts his work week by half. How should the members of the household adjust their consumption expenditures in response to this exogenous shock? It really depends on the severity of the illness affecting the head of the household. If the illness is transitory (a cold, say), then the income earner will be expected to be back on a full-time schedule in a short period of time (within a week, say). In this case, although the family is making no income for one week, there is no reason to implement drastic adjustments in spending patterns. Consumption can go on more or less as usual. The gap between spending and income during the week in which the bread winner of the family is out of commission can be covered with savings accumulated in the past or, if no savings are available, by borrowing a little against future earnings. Future consumption should not be much affected either. For, due to the fact that the period during which income was reduced was short, the interest cost of the borrowing (or decumulation of wealth) that took place during that time is small relative to the level of regular income. However, if the affliction is of a more permanent nature (a chronic back injury, say), then one should expect that the reduction in the work week will be of a permanent nature. In this case, the members of the household should expect not only current but also future income to go down. As a result consumption must be permanently adjusted downward by cutting, for instance, items that are not fully necessary, such as extra school activities or restaurant meals.

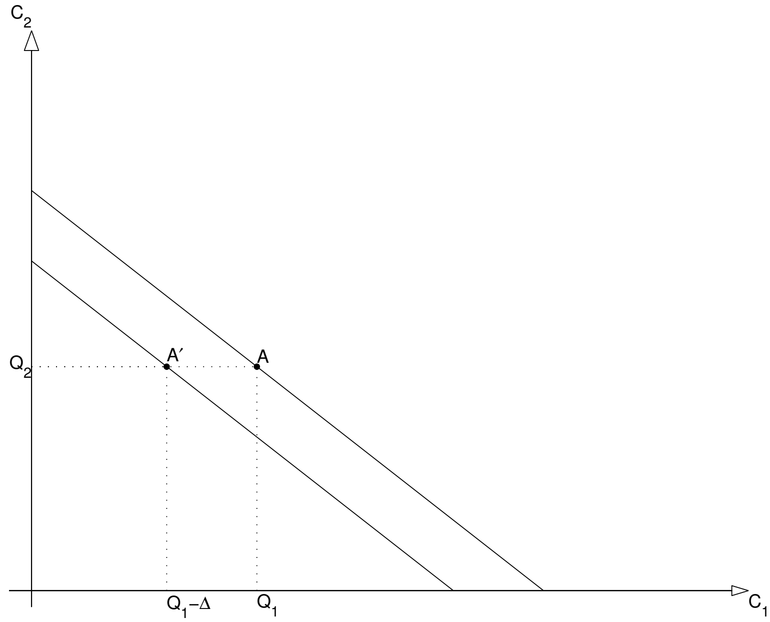
The general principle that the above example illustrates is that forward-looking, optimizing individuals will behave differently in response to an income shock depending on whether it is temporary or permanent. They will tend to finance temporary income shocks, by increasing savings if the temporary shock is positive or by dissaving if the temporary shock is negative. On the other hand, they will adjust in response to permanent income shocks, by cutting consumption if the permanent shock is negative or by increasing consumption if the permanent shock is positive. This same principle can be applied to countries as a whole. In the next two subsections, we develop this principle more formally in the context of our model of current account determination.

3.2.1 Temporary Output Shocks

Consider the adjustment of a small open economy to a temporary variation in output. For example, suppose that Ecuador looses 20 percent of its banana crop due to a drought. Suppose further that this decline in output is temporary, in the sense that it is expected that next year the banana crop will be back at its normal level. How would such a shock affect consumption, the trade balance, and the current account? Intuitively, Ecuadorian households will cope with the negative income shock by running down their savings or even borrowing against their future income levels, which are unaffected by the drought. In this way, they can smooth consumption over time by not having to cut current spending by as much as the decline in current output. It follows that the temporary drought will induce a worsening of the trade balance and the current account.

Formally, assume that the negative shock produces a decline in output in period 1 from Q_1 to $Q_1 - \Delta < Q_1$, but leaves output in period 2 unchanged. The situation is illustrated in figure 3.4, where A indicates the endowment point before the shock (Q_1, Q_2) and A$'$ the endowment point after the shock $(Q_1 - \Delta, Q_2)$. Note that because Q_2 is unchanged points A and A' can be connected by a horizontal line. As a consequence of the decline in Q_1, the budget constraint shifts toward the origin. The new budget constraint is parallel to the old one because the world interest rate is unchanged. The household could adjust to the output shock by reducing consumption in period 1 by exactly the amount of the output decline, Δ, thus leaving consumption in period 2 unchanged. However, if both C_1 and C_2 are normal

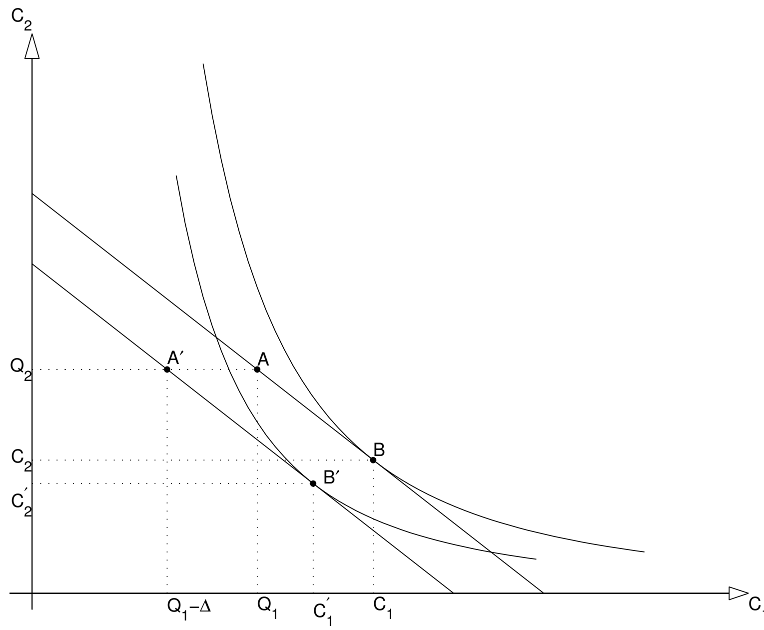
Figure 3.4: A temporary decline in output and the intertemporal budget constraint



goods (i.e., goods whose consumption increases with income), the household will choose to smooth consumption by reducing both C_1 and C_2. Figure 3.5 depicts the economy's response to the temporary output shock. As a result of the shock, the new optimal consumption bundle, B′, is located southwest of the pre-shock consumption allocation, B. In smoothing consumption over time, the country runs a larger trade deficit in period 1 (recall that it was running a trade deficit even in the absence of the shock) and finances it by acquiring additional foreign debt. Thus, the current account deteriorates. In period 2, the country must generate a larger trade surplus than the one it would have produced in the absence of the shock in order to pay back the additional debt acquired in period 1.

The important principle to take away from this example is that tempo-

Figure 3.5: Adjustment to a temporary decline in output

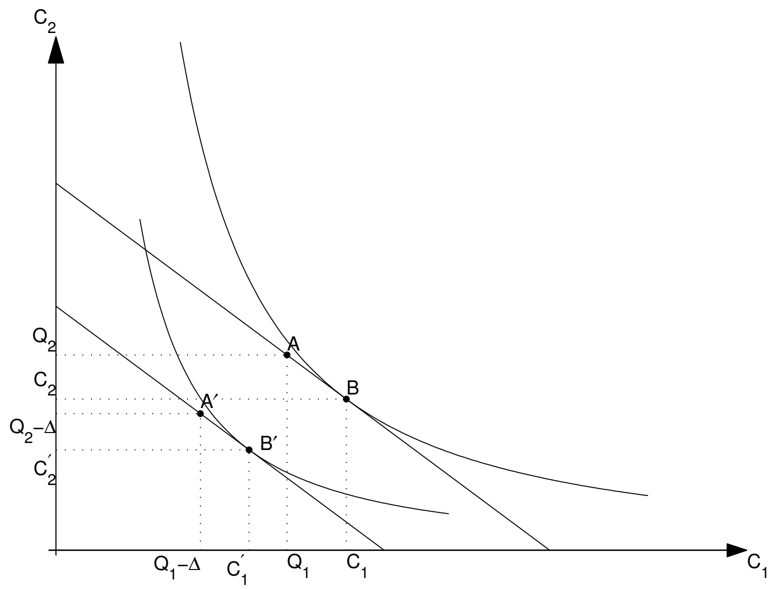


rary negative income shocks are smoothed out by borrowing from the rest of the world rather than by fully adjusting current consumption by the size of the shock. [*Question:* How would the economy respond to a temporary positive income shock?]

3.2.2 Permanent Output Shocks

The pattern of adjustment to changes in income is quite different when the income shock is of a more permanent nature. To continue with the example of the drought in Ecuador, suppose that the drought is not just a one-year event, but is expected to last for many years due to global climate changes. In this case, it would not be optimal for households to borrow against future income, because future income is expected to be as low as

Figure 3.6: Adjustment to a permanent decline in output



current income. Instead, Ecuadorian consumers will have to adjust to the new climatic conditions by cutting consumption in all periods by roughly the size of the decline in the value of the banana harvest.

Formally, consider a permanent negative output shock that reduces both Q_1 and Q_2 by Δ. Figure 3.6 illustrates the situation. As a result of the decline in endowments, the budget constraint shifts to the left in a parallel fashion. The new budget constraint crosses the point $(Q_1 - \Delta, Q_2 - \Delta)$. As in the case of a temporary output shock, consumption-smoothing agents will adjust by reducing consumption in both periods. If consumption in each period fell by exactly Δ, then the trade balance would be unaffected in both periods. In general the decline in consumption should be expected to be close to Δ, implying that a permanent output shock has little consequences for the trade balance and the current account.

Comparing the effects of temporary and permanent output shocks on the current account, the following general principle emerges: Economies will tend to finance temporary shocks (by borrowing or lending on international capital markets) and adjust to permanent ones (by varying consumption in both periods up or down). Thus, temporary shocks tend to produce large movements in the current account while permanent shocks tend to leave the current account largely unchanged.

3.3 Terms-of-Trade Shocks

Thus far, we have assumed that the country's endowments Q_1 and Q_2 can be either consumed or exported. This assumption, although useful to understand the basic functioning of our small open economy, is clearly unrealistic. In reality, the goods that account for most of a country's exports represent only a small fraction of that country's consumers' baskets. For instance, some countries in the Middle East are highly specialized in the production of oil and import a large fraction of the goods they consume. To capture this aspect of the real world, let us now modify our model by assuming that the good households like to consume, say food, is different from the good they are endowed with, say oil. In such an economy, both C_1 and C_2 must be imported, while Q_1 and Q_2 must be exported. Let P^M and P^X denote the prices of imports and exports, respectively. A country's terms of trade, TT, is the relative price of a country's exports in terms of imports, that is, $TT \equiv P^X/P^M$. In terms of our example, TT represents the price of oil in terms of food. Thus, TT indicates the amount of food that the country can buy from the sale of one barrel of oil. Assuming that foreign assets are expressed in units of consumption, the household's budget constraints in periods 1 and 2, respectively, are:

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + TT_1 Q_1$$

and

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + TT_2 Q_2.$$

These budget constraints are identical to (3.1) and (3.2) except for the fact that the terms of trade are multiplying the endowments. Using the terminal condition $B_2^* = 0$, the above two equations can be combined to obtain the following lifetime budget constraint:

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + TT_1 Q_1 + \frac{TT_2 Q_2}{1+r_1}.$$

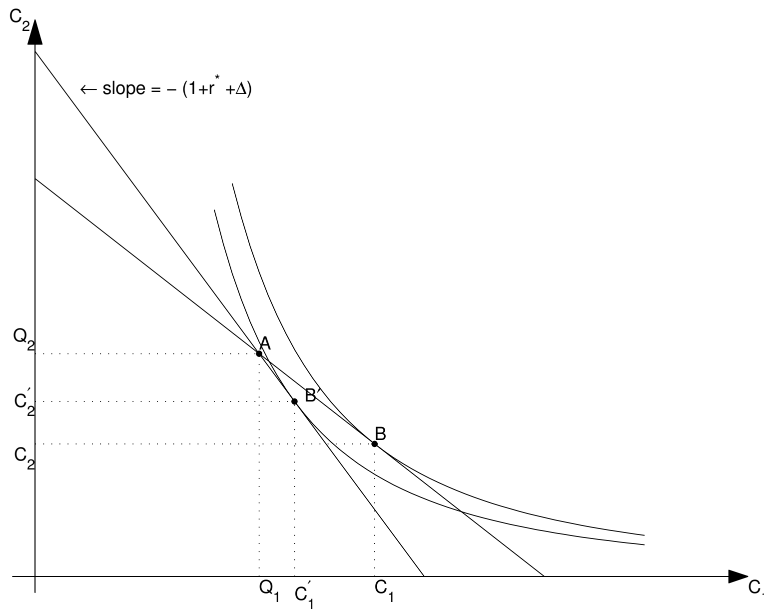
Comparing this lifetime budget constraint with the one given in equation (3.4), it is clear that terms of trade shocks are just like output shocks. Thus, in response to a transitory terms of trade deterioration (a transitory decline in TT), the economy will not adjust consumption much and instead will borrow on the international capital market, which will result in a current account deficit. On the other hand, in response to a permanent terms of trade deterioration (i.e., a fall in both TT_1 and TT_2), the country is likely to adjust consumption down, with little change in the trade balance or the current account.

3.4 World Interest Rate Shocks

An increase in the world interest rate, r^*, has two potentially opposing effects on consumption in period 1. On the one hand, an increase in the interest rate makes savings more attractive because the rate of return on foreign assets is higher. This effect is referred to as the substitution effect, because it induces people to substitute future for present consumption through saving. By the substitution effect, a rise in the interest rate causes consumption in period 1 to decline and therefore the current account to improve. On the other hand, an increase in the interest rate makes debtors poorer and creditors richer. This is called the income effect. By the income effect, an increase in the interest rate leads to a decrease in consumption in period 1 if the country is a debtor, reinforcing the substitution effect, but to an increase in consumption if the country is a creditor, offsetting (at least in part) the substitution effect. We will assume that the substitution effect is stronger than the income effect, so that savings increases in response to an increase in interest rates. Therefore, an increase in the world interest rate, r^*, induces a decline in C_1 and thus an improvement in the trade balance and the current account in period 1.

Figure 3.7 describes the case of an increase in the world interest rate from r^* to $r^* + \Delta$. We deduced before that the slope of the budget constraint is given by $-(1 + r^*)$. Thus, an increase in r^* makes the budget constraint steeper. Because the household can always consume its endowment (recall that B_0^* is assumed to be zero), point A must lie on both the old and the new budget constraints. This means that in response to the increase in r^*,

Figure 3.7: Adjustment to a world interest rate shock



the budget constraint rotates clockwise through point A. The initial optimal consumption point is given by point B, where the household is borrowing in period 1. The new consumption allocation is point B′, which is located west of the original allocation, B. The increase in the world interest rate is associated with a decline in C_1 and thus an improvement in the trade balance and the current account in period 1. Note that because the household was initially borrowing, the income and substitution effects triggered by the rise in the interest rate reinforce each other, so savings increase unambiguously.

3.5 An Economy with Logarithmic Preferences

Thus far, we have used a graphical approach to analyze the determination of the current account in the two-period small open economy. We now illustrate, by means of an example, the basic results using an algebraic approach. Let the utility function be of a log-linear type:

$$U(C_1, C_2) = \ln C_1 + \ln C_2,$$

where ln denotes the natural logarithm. In this case the marginal utility of consumption in the first period, $U_1(C_1, C_2)$, is given by

$$U_1(C_1, C_2) = \frac{\partial U(C_1, C_2)}{\partial C_1} = \frac{\partial(\ln C_1 + \ln C_2)}{\partial C_1} = \frac{1}{C_1}.$$

Similarly, the marginal utility of period 2 consumption, $U_2(C_1, C_2)$ is given by

$$U_2(C_1, C_2) = \frac{\partial U(C_1, C_2)}{\partial C_2} = \frac{\partial(\ln C_1 + \ln C_2)}{\partial C_2} = \frac{1}{C_2}.$$

Here we used the fact that the derivative of the function $\ln x$ is $1/x$, that is, $\partial \ln x/\partial x = 1/x$. The household's first-order condition for utility maximization says that the optimal consumption allocation must satisfy the condition

$$U_1(C_1, C_2) = (1 + r_1)U_2(C_1, C_2).$$

For the logarithmic form of the utility function considered here, the above optimality condition becomes

$$\frac{1}{C_1} = (1 + r_1)\frac{1}{C_2}. \tag{3.8}$$

Next, consider the intertemporal resource constraint of the economy (3.4):

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}.$$

Define $\bar{Y} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$. The variable $\bar{Y}$ represents the present discounted value of the household's total wealth, which is composed of his initial asset holdings and the stream of income (Q_1, Q_2). Note that the household takes $\bar{Y}$ as given. We can rewrite the above expression as

$$C_1 = \bar{Y} - \frac{C_2}{1+r_1}. \tag{3.9}$$

Combining this expression with (3.8), yields

$$C_1 = \frac{1}{2}\bar{Y}.$$

This result says that households find it optimal to consume half of their lifetime wealth in the first half of their lives.

In period 1, the trade balance is the difference between output and domestic spending, or $TB_1 = Q_1 - C_1$, and the current account is the sum of the trade balance and interests received on net foreign assets holdings, or $CA_1 = r_0 B_0^* + TB_1$. Using the definition of $\bar{Y}$ and the fact that in equilibrium the domestic interest rate must equal the world interest rate, or $r_1 = r^*$, we have that C_1, C_2, TB_1, and CA_1 are given by

$$C_1 = \frac{1}{2}\left[(1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r^*}\right] \tag{3.10}$$

$$C_2 = \frac{1}{2}(1+r^*)\left[(1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r^*}\right] \tag{3.11}$$

$$TB_1 = \frac{1}{2}\left[Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r^*}\right] \tag{3.12}$$

$$CA_1 = r_0 B_0^* + \frac{1}{2}\left[Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r^*}\right] \tag{3.13}$$

Consider the effects of temporary and permanent output shocks on the trade balance and the current account. Assume first that income falls temporarily by one unit, that is, Q_1 decreases by one and Q_2 is unchanged. It follows from (3.12) and (3.13) that the trade balance and the current account both fall by half a unit. This is because consumption in period 1 falls by only half a unit.

Suppose next that income falls permanently by one unit, that is, Q_1 and Q_2 both fall by one. Then the trade balance and the current account decline by $\frac{1}{2}\frac{r^*}{1+r^*}$. Consumption in period 1 falls by $\frac{1}{2}\frac{2+r^*}{1+r^*}$. For realistic values of r^*, the predicted deterioration in the trade balance and current account in response to the assumed permanent negative income shock is close to zero and in particular much smaller than the deterioration associated with the temporary negative income shock. For example, assume that the world interest rate is 10 percent, $r^* = 0.1$. Then, both the trade balance and the current account in period 1 fall by 0.046 in response to the permanent output shock and by 0.5 in response to the temporary shock. That is, the current account deterioration is 10 times larger under a temporary shock than under a permanent one.

Finally, consider the effect of an increase in the world interest rate r^*.

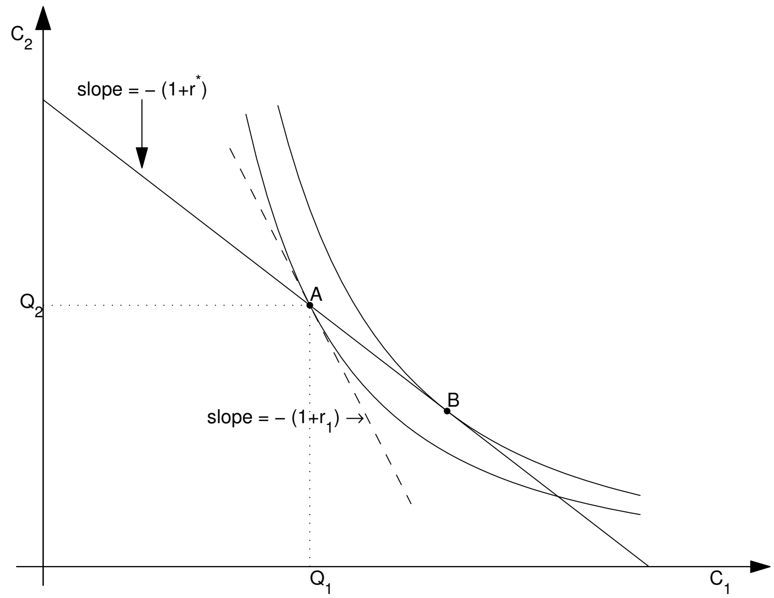
Clearly, in period 1 consumption falls and both the trade balance and the current account improve. Note that the decline in consumption in period 1 is independent of whether the country is a net foreign borrower or a net foreign lender in period 1. This is because for the particular preference specification considered in this example, the substitution effect always dominates the income effect.

3.6 Capital Controls

Current account deficits are often viewed as bad for a country. The idea behind this view is that by running a current account deficit the economy is living beyond its means. As a result, the argument goes, as the country accumulates external debt, it imposes future economic hardship on itself in the form of reduced consumption and investment spending when the foreign debt becomes due. A policy recommendation frequently offered to countries undergoing external imbalances is the imposition of capital controls. In their most severe form, capital controls consist in the prohibition of borrowing from the rest of the world. Milder versions take the form of taxes on international capital inflows.

We can use the model economy developed in this chapter to study the welfare consequences of prohibiting international borrowing. Suppose that the equilibrium under free capital mobility is as depicted in figure 3.8. The optimal consumption basket is given by point B and the endowment bundle is represented by point A. The figure is drawn under the assumption that the economy starts period 1 with a nil asset position ($B_0^* = 0$). In the uncon-

Figure 3.8: Equilibrium under capital controls



strained equilibrium, households optimally choose to borrow from the rest of the world in period 1 in order to finance a level of consumption that exceeds their period-1 endowment. As a result, in period 1 the trade balance (TB_1), the current account (CA_1), and the net foreign asset position (B_1^*) are all negative. In period 2, consumption must fall short of period-2 endowment to allow for the repayment of the debt contracted in period 1 plus the corresponding interest. Assume now that the government prohibits international borrowing. That is, the policymaker imposes financial restrictions under which B_1^* must be greater than or equal to zero. Agents cannot borrow from the rest of the world in period 1, therefore their consumption can be at most as large as their endowment. It is clear from figure 3.8 that any point on the intertemporal budget constraint containing less period-1 consumption than at point A (i.e., any point on the budget constraint located northwest of A) is less preferred than point A. This means that when the capital controls are imposed, households choose point A, and the borrowing constraint is binding. In the constrained equilibrium we have that $B_1^* = 0$ and $C_1 = Q_1$. The fact that consumption equals the endowment implies that the trade balance in period 1 is zero ($TB_1 = Q_1 - C_1 l = 0$). Further, given our assumption that the initial net foreign asset position is zero ($B_0^* = 0$), the current account in period 1 is also nil ($CA_1 = TB_1 + r_0 B^* 0 = 0$). This in turn implies that the country starts period 2 with zero external debt ($B_1^* = B_0^* + CA_1 = 0$). As a consequence, the country can sepnd its entire period-2 endowment in consumption ($C_2 = Q_2$).

The capital controls are successful in achieving the government's goal of curbing current-account deficits and allowing for higher future spending. But do capital controls make households happier? To answer this question, note that the indifference curve that passes through the endowment point A, the consumption bundle under capital controls, lies southwest of the indifference curve that passes through point B, the optimal consumption bundle under free capital mobility. Therefore, capital controls lower the level of utility, or welfare.

Under capital controls the domestic interest rate r_1 is no longer equal to the world interest rate r^*. At the world interest rate, domestic households would like to borrow from foreign lenders in order to spend beyond their endowments. But capital controls make international funds unavailable. Thus, the domestic interest rate must rise above the world interest rate to bring about equilibrium in the domestic financial market. Graphically, $1+r_1$

is given by the negative of the slope of the indifference curve at point A. The slope at this point is given by the slope of the dashed line in figure 3.8. Only at that interest rate are households willing to consume exactly their endowment. Formally, the domestic interest rate under capital controls is the solution to the following expression:

$$U_1(Q_1, Q_2) = (1 + r_1)U_2(Q_1, Q_2).$$

This expression is the household's optimality condition for the allocation of consumption over time, evaluated at the endowment point. Notice that because Q_1 and Q_2 are exogenously given, the above expression represents one equation in one unknown, r_1. The smaller is Q_1 relative to Q_2, the higher will be the marginal utility of consumption today relative to the marginal utility of consumption next period, and therefore the higher will be the interest rate. Intuitively, the lower is output in period 1 relative to output period 2, all other things equal, the larger will be the desire to borrow in period 1. To dissuade agents from borrowing in period 1, the domestic interest rate must rise.

3.7 Exercises

1. Consider a two-period small open endowment economy populated by a large number of households with preferences described by the lifetime utility function

$$C_1^{\frac{1}{10}} C_2^{\frac{1}{11}},$$

where C_1 and C_2 denote, respectively, consumption in periods 1 and 2. Suppose that households receive exogenous endowments of goods given by $Q_1 = Q_2 = 10$ in periods 1 and 2, respectively. Every household enters period 1 with some debt, denoted B_0^*, inherited from the past. Let B_0^* be equal to -5. The interest rate on these liabilities, denoted r_0, is 20 percent. Finally, suppose that the country enjoys free capital mobility and that the world interest rate on assets held between periods 1 and 2, denoted r^*, is 10 percent.

   (a) Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1 and 2.

   (b) Assume now that the endowment in period 2 is expected to increase from 10 to 15. Calculate the effect of this anticipated output increase on consumption, the trade balance, and the current account in both periods. Compare your answer to that you gave for item (a) and provide intuition.

   (c) Finally, suppose now that foreign lenders decide to forgive all of the country's initial external debt. How does this decision affect the country's levels of consumption, trade balance, and current account in periods 1 and 2. (For this question, assume that $Q_1 = Q_2 = 10$.) Compare your answer to the one you gave for item (a) and explain.

2. **The Terms of Trade and the Current Account**

Consider the following chart showing commodity prices in world markets:

	Price	
Commodity	Period 1	Period2
Wheat	1	1
Oil	1	2

In the table, prices of oil are expressed in dollars per barrel, prices of wheat are expressed in dollars per bushel. Kuwait is a two-period economy that produces oil and consumes wheat. Consumers have preferences described by the lifetime utility function

$$U(C_1, C_2) = C_1 \times C_2,$$

where C_1 and C_2 denote, respectively, consumption of wheat in periods 1 and 2, measured in bushels. Kuwait's per-capita endowments of oil are 5 barrels in each period. The country starts period 1 with net financial assets carried over from period 0, including interest of 10 percent, worth 1.1 bushels of wheat (i.e., $(1 + r_0)B_0^* = 1.1$). The interest rate in period 0 is assumed to be 10 percent (i.e., $r_0 = 0.1$).

The country enjoys free capital mobility and the world interest rate is 10 percent. Financial assets are denominated in units of wheat.

(a) What are the terms of trade faced by Kuwait in periods 1 and 2?

(b) Calculate consumption, the trade balance, the current account and national savings in periods 1 and 2.

(c) Answer the previous question assuming that the price of oil in the second period is not 2 but 1 dollar per barrel. Compare your answers to this and the previous question and provide intuition.

3. **Capital controls.** Consider a two-period model of a small open economy with a single good each period and no investment. Let preferences of the representative household be described by the utility function

$$U(C_1, C_2) = \sqrt{C_1} + \beta\sqrt{C_2}$$

The parameter β is known as the subjective discount factor. It measures the consumer's degree of impatience in the sense that the smaller is β, the higher is the weight the consumer assigns to present consumption relative to future consumption. Assume that $\beta = 1/1.1$. The representative household has initial net foreign wealth of $(1 + r_0)B_0^* = 1$, with $r_0 = 0.1$, and is endowed with $Q_1 = 5$ units of goods in period 1 and $Q_2 = 10$ units in period 2. The world interest rate paid on assets held from period 1 to period 2, r^*, equals 10% (i.e., $r^* = 0.1$) and there is free international capital mobility.

(a) Calculate the equilibrium levels of consumption in period 1, C_1, consumption in period 2, C_2, the trade balance in period 1, TB_1, and the current account balance in period 1, CA_1.

(b) Suppose now that the government imposes capital controls that require that the country's net foreign asset position at the end of period 1 be nonnegative ($B_1^* \geq 0$). Compute the equilibrium value of the domestic interest rate, r_1, consumption in periods 1 and 2, and the trade and current account balances in period 1.

(c) Evaluate the effect of capital controls on welfare. Specifically, find the level of utility under capital controls and compare it to the level of utility obtained under free capital mobility.

(d) For this question and the next, suppose that the country experiences a temporary increase in the endowment of period 1 to $Q_1 = 9$, with period 2 endowment unchanged. Calculate the effect of this output shock on C_1, C_2, TB_1, CA_1, and r_1 in the case that capital is freely mobile across countries.

(e) Finally, suppose that the capital controls described in part (b) are in place. Will they still be binding (i.e., affect household behavior)?

4. Consider the equilibrium with capital controls analyzed in section3.6 and depicted in figure 3.8. Suppose the equilibrium allocation under free capital mobility is at a point on the intertemporal budget constraint located northwest of the endowment point A. Suppose that capital controls prohibit borrowing or lending intenationally. Would it still be true that capital controls are welfare decreasing?

Chapter 4

Uncertainty and the Current Account

Thus far, we have studied the response of the current account to changes in fundamentals that are known with certainty. The real world, however, is an uncertain place. Some periods display higher macroeconomic volatility than others. A natural question, therefore, is how the overall level of uncertainty affects the macroeconomy, and, in particular, the external accounts. This chapter is devoted to addressing this question. It begins by documenting a period of remarkable stability in the United states, known as the Great Moderation. It then shows that this period coincided with the emergence of large current account deficits. Finally, the chapter expands the small open economy model of chapter 3 to allow for uncertainty. This modification allows us to understand the effect of changes in the aggregate level of uncertainty on consumption, savings, the trade balance, and the current account.

4.1 The Great Moderation

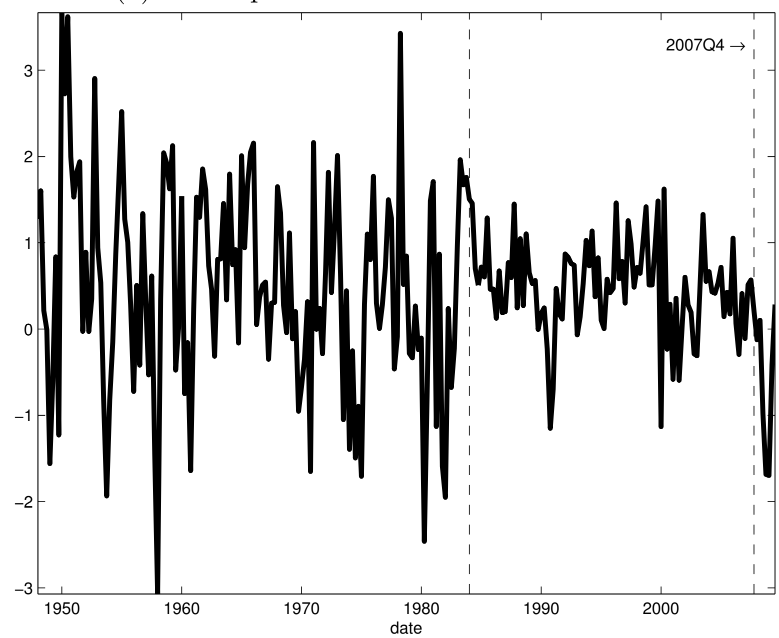
A number of researchers have documented that the volatility of U.S. output declined significantly starting in the early 1980s. This phenomenon has become known as the Great Moderation.[1] The most commonly used measure of volatility in macroeconomic data is the standard deviation. According to this statistic, U.S. output growth became half as volatile in the past quarter century. Specifically, the standard deviation of quarter-to-quarter output growth was 1.2 percent over the period 1948 to 1983, but only 0.5 percent over the period 1984 to 2006. Panel (a) of figure 4.1 depicts the quarterly growth rate of U.S. output from 1948:Q1 to 2009:Q3. It also shows with a vertical line the beginning of the Great Moderation in 1984. It is evident from the figure that the time series of output growth in the United States is much smoother in the post 1984 subsample than it is in the pre-1984 subsample.

Researchers have put forward three alternative explanations of the Great Moderation: good luck, good policy, and structural change. The good-luck hypothesis states that by chance, starting in the early 1980s the U.S. economy has been blessed with smaller shocks. The good policy hypothesis maintains that starting with former Fed chairman Paul Volker's aggressive monetary policy that brought to an end the high inflation of the

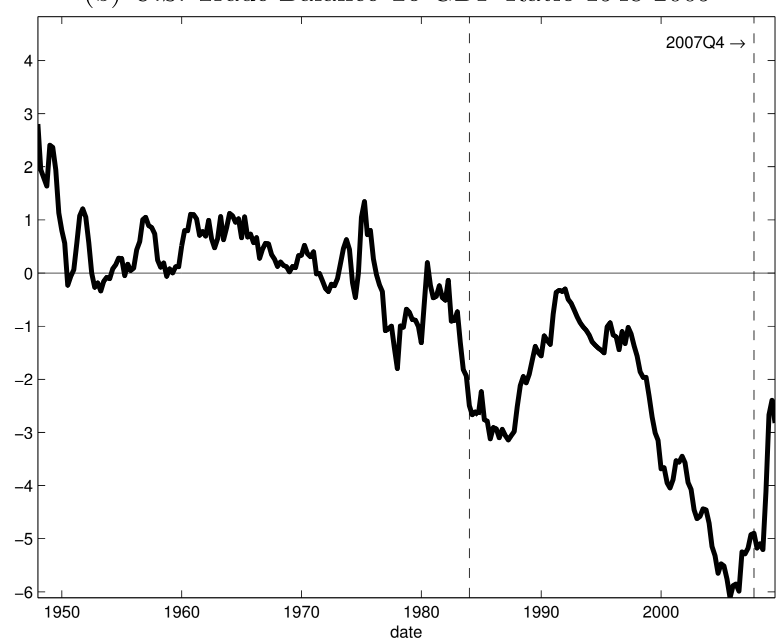
[1] Early studies documenting the Great Moderation are Kim and Nelson (1999) and McConnell and Perez-Quiróz (2000). Stock and Watson (2002) present a survey of this literature.

Figure 4.1: The Great Moderation

(a) Per Capita U.S. GDP Growth 1948-2009



(b) U.S. Trade Balance To GDP Ratio 1948-2009



Source: `http://wow.bea.gov`

1970s and continuing with the low inflation policy of Volker's successor Alan Greenspan, the United States experienced a period of extraordinary macroeconomic stability. Good regulatory policy has also been credited with the causes of the Great Moderation. Specifically, the early 1980s witnessed the demise of regulation Q (or Reg Q). Regulation Q imposed a ceiling on the interest rate that banks could pay on deposits.[2] As a result of this financial distortion, when expected inflation goes up (as it did in the 1970s) the real interest rate on deposits falls and can even become negative, inducing depositors to withdraw their funds from banks. As a consequence, banks are forced to reduce the volume of loans generating a credit-crunch-induced recession. The third type of explanation states that the Great Moderation was in part caused by structural change, particularly in inventory management and in the financial sector.

4.1.1 The Great Moderation And The Emergence of Trade Imbalances

We will not dwell on which of the proposed explanations of the Great Moderation has more merit. Instead, our interest is in possible connections between the Great Moderation and the significant trade balance deterioration observed in the U.S. over the period 1984 to 2006. Panel (b) of figure 4.1 displays the ratio of the trade balance to GDP in the United States over

[2]Regulation Q became law in 1933. Its objective was to make banks more stable. Competition for deposits was thought to increases costs for banks and to force them into making riskier loans with higher expected returns. Thus allowing banks to pay interest on deposits was believed contribute to bank failures. For more information on Reg Q see R. Alton Gilbert, "Requiem for Regulation Q: What It Did and Why It Passed Away," Federal Reserve Bank of St. Louis Review, February 1986, 68(2), pp. 22-37.

the period 1948-2009. During the period 1948-1984 the United States experienced on average positive trade balances of about 0.2 percent of GDP. Starting in the early 1980s, however, the economy was subject to a string of large trade deficits averaging 2.6 percent of GDP.

Is the timing of the Great Moderation and the emergence of protracted trade deficits pure coincidence, or is there a causal connection between the two? To address this issue, we will explore the effects of changes in output uncertainty on the trade balance and the current account in the context of our theoretical framework of current account determination.

4.2 A Model With Uncertainty

In the economy studied in chapter 3, the endowments Q_1 and Q_2 are known with certainty. What would be the effect of making the future endowment, Q_2, uncertain? That is, how would households adjust their consumption and savings decisions in period 1 if they knew that the endowment in period 2 could be either high or low with some probability? Intuitively, we should expect the emergence of precautionary savings in period 1. That is, an increase in savings in period 1 to hedge against a bad income realization in period 2. The desired increase in savings in period 1 must be brought about by a reduction in consumption in that period. With period-1 endowment unchanged and consumption lower, the trade balance must improve. We therefore have that an increase in uncertainty brings about an improvement in the trade balance. By the same token, a decline in income uncertainty, such as the one observed in the United States since the early 1980s, should be associated with a deterioration in the trade balance.

To formalize these ideas, consider an economy in which initially, the stream of output is known with certainty and constant over time. Specifically suppose that $Q_1 = Q_2 = Q$. Assume further that preferences are of the form $\ln C_1 + \ln C_2$. To simplify the analysis, assume that initial asset holdings are nil, that is, $B_0^* = 0$, and that the world interest rate is nil, or $r^* = 0$. In this case, the intertemporal budget constraint of the representative household is given by $C_2 = 2Q - C_1$. Using this expression to eliminate C_2 from the utility function, we have that the household's utility maximization problem consists in choosing C_1 so as to maximize $\ln C_1 + \ln(2Q - C_1)$. The solution to this problem is $C_1 = C_2 = Q$. It follows that the trade balance in period 1, given by $Q_1 - C_1$, is zero. That is,

$$TB_1 = 0.$$

In this economy households do not need to save or dissave in order to smooth consumption over time because the endowment stream is already perfectly smooth.

Consider now a situation in which Q_2 is not known with certainty in period 1. Specifically, assume that with probability 1/2 the household receives a positive endowment shock in period 2 equal to $\sigma > 0$, and that with equal probability the household receives a negative endowment shock in the amount of $-\sigma$. That is,

$$Q_2 = \begin{cases} Q + \sigma & \text{with probability } 1/2 \\ Q - \sigma & \text{with probability } 1/2 \end{cases}.$$

We continue to assume that $Q_1 = Q$. Note that this is a mean-preserving increase in period-2 income uncertainty in the sense that the expected value of the endowment in period 2, given by $\frac{1}{2}(Q+\sigma) + \frac{1}{2}(Q-\sigma)$ equals Q, which equals the endowment that the household receives in period 2 in the economy without uncertainty.

The standard deviation of the endowment in period 2 is given by σ. To see this, recall that the standard deviation is the square root of the variance and that, in turn, the variance is the expected value of the deviation of output from its mean. The deviation of output from its mean is $Q+\sigma-Q = \sigma$ in the high-output state and $Q - \sigma - Q = -\sigma$ in the low-output state. Therefore, the variance of output in period 2 is given by $\frac{1}{2} \times \sigma^2 + \frac{1}{2} \times (-\sigma)^2 = \sigma^2$. The standard deviation of period-2 output is then given by $\sqrt{\sigma^2} = \sigma$. It follows that the larger is σ the more volatile is the period-2 endowment.

We must specify how households value uncertain consumption bundles. We will assume that households care about the expected value of utility. Specifically, preferences under uncertainty are given by

$$\ln C_1 + E \ln C_2,$$

where E denotes expected value. Note that this preference formulation encompasses the preference specification we used in the absence of uncertainty.

This is because when C_2 is known with certainty, then $E \ln C_2 = \ln C_2$.

The budget constraint of the household in period 2 is given by $C_2 = 2Q + \sigma - C_1$ in the good state of the world and by $C_2 = 2Q - \sigma - C_1$ in the bad state of the world. Therefore, expected lifetime utility, $\ln C_1 + E \ln C_2$, is given by

$$\ln C_1 + \frac{1}{2}\ln(2Q + \sigma - C_1) + \frac{1}{2}\ln(2Q - \sigma - C_1).$$

The household chooses C_1 to maximize this expression. The first-order optimality condition associated with this problem is

$$\frac{1}{C_1} = \frac{1}{2}\left[\frac{1}{2Q + \sigma - C_1} + \frac{1}{2Q - \sigma - C_1}\right] \tag{4.1}$$

The left-hand side of this expression is the marginal utility of consumption in period 1, or $U_1(C_1, C_2)$. The right-hand side is the expected value of the marginal utility of consumption in peirod 2, or $EU_2(C_1, C_2)$. This means that the household consumpiton choice equates the marginal utility of consumption in period 1 to the expected marginal utility of consumption in period 2, or $EU_2(C_1, C_2)$.

Consider first whether the optimal consumption choice associated with the problem without uncertainty, given by $C_1 = Q$, represents a solution in the case with uncertainty. If this was the case, then it would have to be true that

$$\frac{1}{Q} = \frac{1}{2}\left[\frac{1}{2Q + \sigma - Q} + \frac{1}{2Q - \sigma - Q}\right].$$

This expression can be further simplified to

$$\frac{1}{Q} = \frac{1}{2}\left[\frac{1}{Q+\sigma} + \frac{1}{Q-\sigma}\right].$$

Further simplifying, we obtain

$$1 = \frac{Q^2}{Q^2 - \sigma^2},$$

which is impossible, given that $\sigma > 0$. We have shown that if we set $C_1 = Q$, then the left side of optimality condition (4.1) is less than the right side. In other words, if the consumer chose $C_1 = Q$, then the marginal utility of consumption in period 1 would be smaller than the expected marginal utility of consumption in period 2. It follows that the household would be better off consuming less in period 1 and more in period 2. Formally, because the left side of optimality condition (4.1) is decreasing in C_1 whereas the right side is increasing in C_1, it must be the case that the optimal level of consumption in period 1 satisfies

$$C_1 < Q.$$

It then follows that in the economy with uncertainty the trade balance is positive in period 1, or

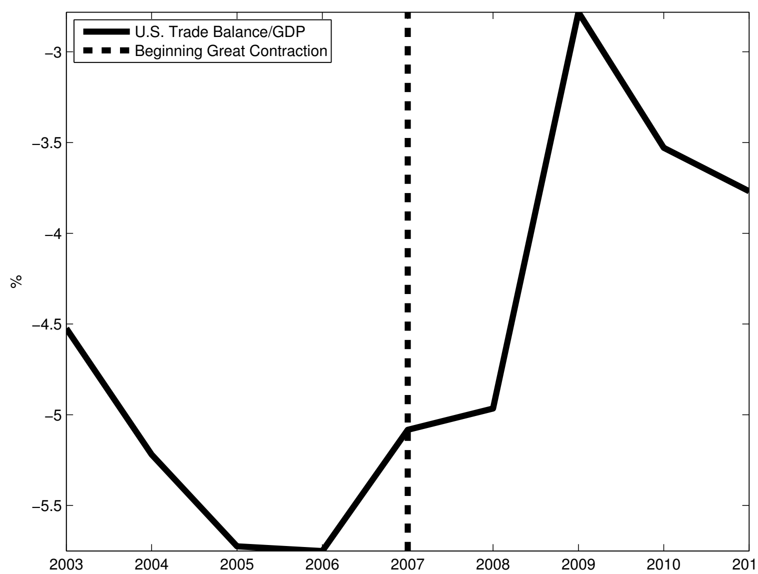
$$TB_1 > 0.$$

Households use the trade balance as a vehicle to save in period 1. In this way, they avoid having to cut consumption by too much in the bad state of period 2. The reason for this behavior is that with a convex marginal utility of consumption in period 2 a gift of σ units of consumption reduces marginal utility by less than the increase in marginal utility caused by a loss of consumption in the amount of σ units. As a result, the prospect of consuming $Q + \sigma$ or $Q - \sigma$ with equal probability in period 2 increases the expected marginal utility of consumption in that period. Because at the optimum, today's marginal utility must equal next period's in expected value, and because current marginal utility is decreasing in current consumption, the adjustment to a mean-preserving increase in uncertainty about next period's endowment takes the form of a reduction in current consumption.

4.3 The Return of Uncertainty: The Great Contraction And The Current Account

The model presented in this chapter captures qualitatively the joint occurrence of diminished output uncertainty and trade deficits observed during the Great Moderation (1984-2006). A natural test of the model is whether the elevated level of aggregate uncertainty the U.S. economy has been experiencing since the onset of the Great Contraction of 2007 has been accompanied by an improvement in the trade balance. It turns out that this is indeed the case. Look at figure 4.2. It displays the U.S. current account balance as a percentage of GDP between 2003 and 2011. Over the four years preceeding the Great Contraction, 2003-2006, the average U.S. trade balance deficit was about 5.4 percent of GDP. Over the four years since the onset of the Great Contraction, 2007-2011, the trade balance deficit was on average 3.4 percent of GDP. This means that the crisis era was associated

Figure 4.2: The Great Contraction And The Trade Balance



Source: `http://www.bea.gov`

with an improvement in the trade balance of 2 percent of output.

4.4 Exercises

1. **[Risk Neutrality]** Redo the analysis in section 4.1 assuming that households are risk neutral in period 2. Specifically, assume that their preferences are logarithmic in period-1 but linear in period-2 consumption. What would be the predicted effect of the Great Moderation on the trade balance in period 1?

2. **[Certainty Equivalence]** Consider a two-period, small, open, endowment economy populated by households with preferences described by the utility function given by

$$-\frac{1}{2}(C_1 - \bar{C})^2 - \frac{1}{2}E(C_2 - \bar{C})^2,$$

where $\bar{C}$ represents a satiation level of consumption, and E denotes the mathematical expectations operator. In period 1, households receive an endowment $Q_1 = 1$ and have no assets or liabilities carried over from the past ($B_0^* = 0$). Households can borrow or lend in the international financial market at the world interest rate $r^* = 0$. Compute consumption and the trade balance in periods 1 and 2 under the following two assumptions regarding the endowment in period 2, denoted Q_2: (a) Q_2 equals 1; and (b) Q_2 is random and takes the values 0.5 with probability 1/2 or 1.5 with probability 1/2. Provide intuition for your findings.

3. **[The Current Account As Insurance Against Catastrophic Events]** Consider a two-period endowment economy populated by

identical households with preferences defined over consumption in period 1, C_1 and consumption in period 2, C_2, and described by the utility function

$$\ln C_1 + E \ln C_2,$$

where C_1 denotes consumption in period 1, C_2 denotes consumption in period 2, and E denotes the expected value operator. Each period, the economy receives an endowment of 10 units of food. Households start period 1 carrying no assets or debts from the past. The interest rate on financial assets held between periods 1 and 2 is zero.

(a) Compute consumption, the trade balance, the current account, and national savings in period 1.

(b) Assume now that the endowment in period 1 continues to be 10, but that the economy is prone to severe natural disasters in period 2. Suppose that these negative events are very rare, but have catastrophic effects on the country's output. Specifically, assume that with probability 0.01 the economy suffers an earthquake in period 2 that causes the endowment to drop by 90 percent with respect to period 1. With probability 0.99, the endowment in period 2 increases to 111/11.

   i. What is the expected endowment in period 2? How does it compare to that of period 1?

   ii. What percent of period-1 endowment will the country export? Compare this answer to what happens under certainty and provide intuition.

Chapter 5

Current Account Determination in a Production Economy

Thus far, we have considered an endowment economy without investment, so that the current account was simply determined by savings. In this chapter, we extend our theory by studying the determination of the current account in an economy with investment in physical capital. In this economy, output is not given exogenously, but is instead produced by firms.

5.1 A production economy

5.1.1 Firms

Consider an economy in which output is produced with physical capital. Specifically, let K_1 and K_2 denote the capital stocks at the beginning of periods 1 and 2, respectively, and assume that output is an increasing function of capital. Formally,

$$Q_1 = F(K_1)$$

and

$$Q_2 = F(K_2),$$

where, as before, Q_1 and Q_2 denote output in periods 1 and 2. $F(\cdot)$ is a production function, that is, a technological relation specifying the amount of output obtained for each level of capital input. Output is assumed to be zero when the capital stock is zero ($F(0) = 0$). We also assume that output is increasing in capital. Another way of stating this assumption is to say that the marginal product of capital is positive. The marginal product of capital is the amount by which output increases when the capital stock is increased by one unit and is given by the derivative of the production function with respect to capital:

$$\text{marginal product of capital } = F'(K).$$

Finally, we assume that the marginal product of capital is decreasing in K, that is, $F''(K) < 0$, which implies that the production function is concave.

Panel (a) of figure 5.1 displays output as a function of the capital stock. The marginal product of capital at $K = K^*$, $F'(K^*)$, is given by the slope of $F(K)$ at $K = K^*$. Panel (b) of figure 5.1 displays the marginal product of capital as a function of K.

Output is produced by firms. In period 1, the capital stock K_1 is predetermined, and thus so is output, Q_1. To produce in period 2 firms must borrow capital in period 1 at the interest rate r_1. Physical capital depreciates at the rate δ between periods 1 and 2. Therefore, the total cost of borrowing one unit of capital in period 1 is $r_1+\delta$. Profits in period 2, Π_2, are then given by the difference between output and the rental cost of capital, that is

$$\Pi_2 = F(K_2) - (r_1 + \delta)K_2. \tag{5.1}$$

Firms choose K_2 so as to maximize profits, taking as given the interest rate r_1. Figure 5.2 displays the level of capital that maximizes profits. For values of K below K_2, the marginal product of capital exceeds the rental cost $r_1+\delta$, thus, the firm can increase profits by renting an additional unit of capital. For values of K greater than K_2, the rental cost of capital is greater than the marginal product of capital, so the firm can increase profits by reducing K. Therefore, the optimal level of capital, is the one at which the marginal product of capital equals the rental cost of capital, that is,[1]

$$F'(K_2) = r_1 + \delta \tag{5.2}$$

[1]Equation (5.2) is in fact the first-order necessary condition for profit maximization. To see why, take the derivative of the right-hand side of (5.1) with respect to K_2 and equate it to zero.

Figure 5.1: The production function, $F(K)$

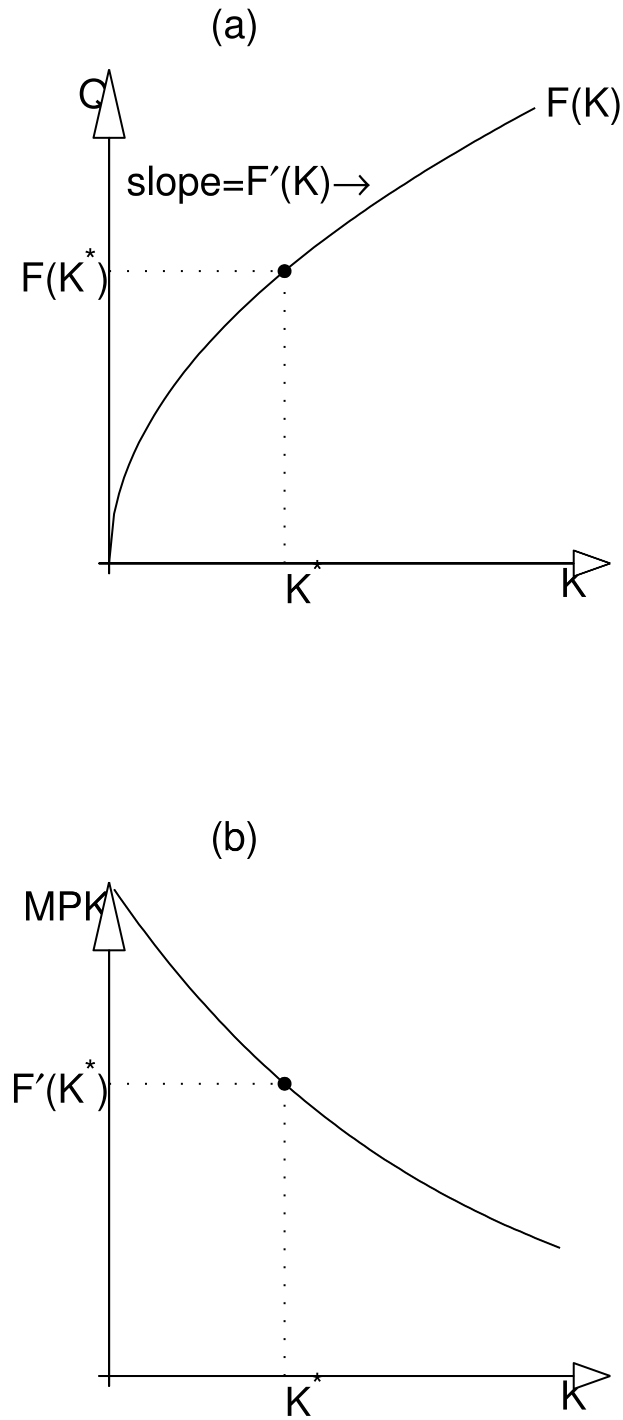
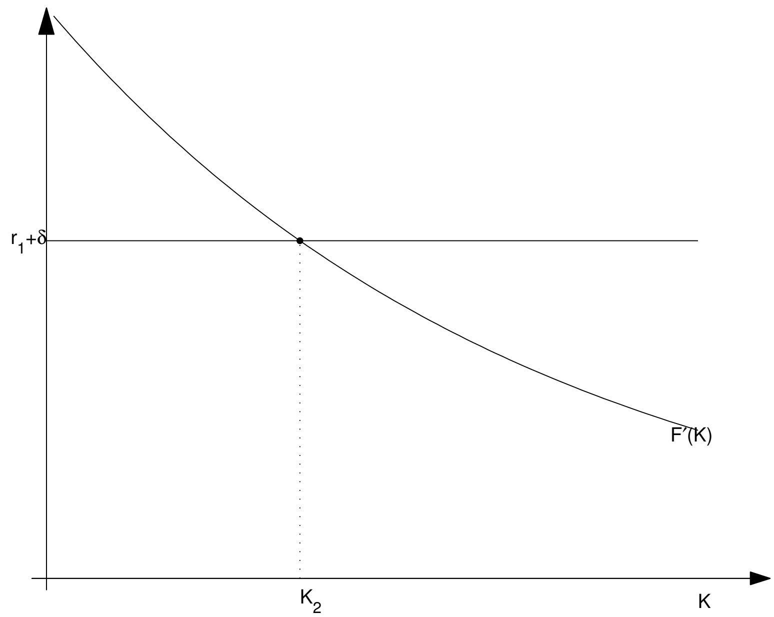


Figure 5.2: Marginal product and marginal cost schedule



Because the marginal product of capital is decreasing in the level of the capital stock, it follows from equation (5.2) that K_2 is a decreasing function of r_1. Intuitively, as r_1 goes up so does the rental cost of capital, so firms choose to hire fewer units of this factor input.

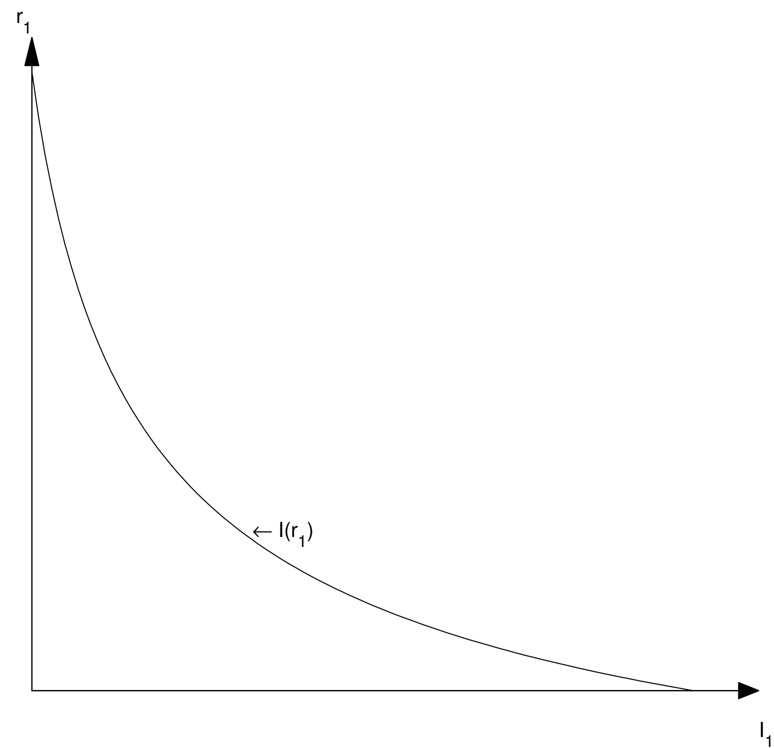
Investment in physical capital in period 1, I_1, is defined as the difference between the capital stock in period 2 and the undepreciated part of the capital stock in period 1,[2]

$$I_1 = K_2 - (1 - \delta)K_1 \tag{5.3}$$

Because K_1 is a predetermined variable in period 1, it follows that, given K_1

[2]Strictly speaking, I_t is called *gross investment* and is equal to the sum of *net investment*, $K_2 - K_1$, which measures the increase in the capital stock, and depreciation, δK_1.

Figure 5.3: The investment schedule, $I(r)$



and δ, I_1 moves one for one with K_2. Thus, I_1 is a decreasing function of r_1. Figure 5.3 depicts the relationship between the interest rate and investment demand in period 1, holding constant K_1 and δ.

In period 1, profits are given by the difference between output, $F(K_1)$, and the rental cost of capital, $(r_0 + \delta)K_1$, that is,

$$\Pi_1 = F(K_1) - (r_0 + \delta)K_1, \tag{5.4}$$

As we mentioned above, the initial capital stock K_1 is given. Therefore, period 1 profits are also given.

5.1.2 Households

Consider now the behavior of households. At the beginning of period 1, the household is endowed with W_0 units of interest bearing wealth. The rate of return on wealth is given by r_0. Thus, interest income is given by $r_0 W_0$. In addition, the household is the owner of the firm and thus receives the firm's profits, Π_1. Therefore, total household income in period 1 equals $r_0 W_0 + \Pi_1$. As in the endowment economy, the household uses its income for consumption and additions to the stock of wealth. The budget constraints of the household in period 1 is then given by

$$C_1 + (W_1 - W_0) = r_0 W_0 + \Pi_1 \tag{5.5}$$

Similarly, the household's budget constraint in period 2 takes the form:

$$C_2 + (W_2 - W_1) = r_1 W_1 + \Pi_2, \tag{5.6}$$

where W_2 denotes the stock of wealth the household chooses to hold at the end of period 2. Because period 2 is the last period of life, the household will not want to hold any positive amount of assets maturing after that period. Consequently, the household will always find it optimal to choose $W_2 \leq 0$. At the same time, the household is not allowed to end period 2 with unpaid debts (the no-Ponzi-game condition), so that $W_2 \geq 0$. Therefore, household's wealth at the end of period 2 must be equal to zero:

$$W_2 = 0.$$

Using this expression, the budget constraint (5.6) becomes

$$C_2 = (1 + r_1)W_1 + \Pi_2. \tag{5.7}$$

Combining (5.5) and (5.7) to eliminate W_1 yields the following intertemporal budget constraint of the household:

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)W_0 + \Pi_1 + \frac{\Pi_2}{1 + r_1} \tag{5.8}$$

This expression is similar to the intertemporal budget constraint corresponding to the endowment economy, equation (3.4), with the only difference that the present discounted value of lifetime endowments is replaced by the present discounted value of profits. As in the endowment economy, households derive utility from consumption in periods 1 and 2. Their preferences are described by the utility function (3.5), which we reproduce here for convenience:

$$U(C_1, C_2).$$

The household chooses C_1 and C_2 so as to maximize the utility function subject to the intertemporal budget constraint (5.8) taking as given Π_1, Π_2, $(1+r_0)W_0$, and r_1. The household's maximization problem is identical to the one we discussed in the endowment economy. In particular, at the optimal consumption basket, the indifference curve is tangent to the intertemporal budget constraint. That is, the slope of the indifference curve is equal to $-(1 + r_1)$.

Before studying the determination of the current account, it is instructive to analyze a closed economy, that is, an economy in which agents do not have access to international financial markets, so that the current account is always zero.

5.1.3 Equilibrium in a closed economy

In a closed economy, agents do not have access to the world capital market. As a consequence, the household's wealth must be held in the form of claims to domestic capital, that is

$$W_0 = K_1$$

and

$$W_1 = K_2.$$

Replacing Π_1 with (5.4), Π_2 with (5.1), $F(K_1)$ with Q_1, and $F(K_2)$ with Q_2, equations (5.5) and (5.7) can be written as:

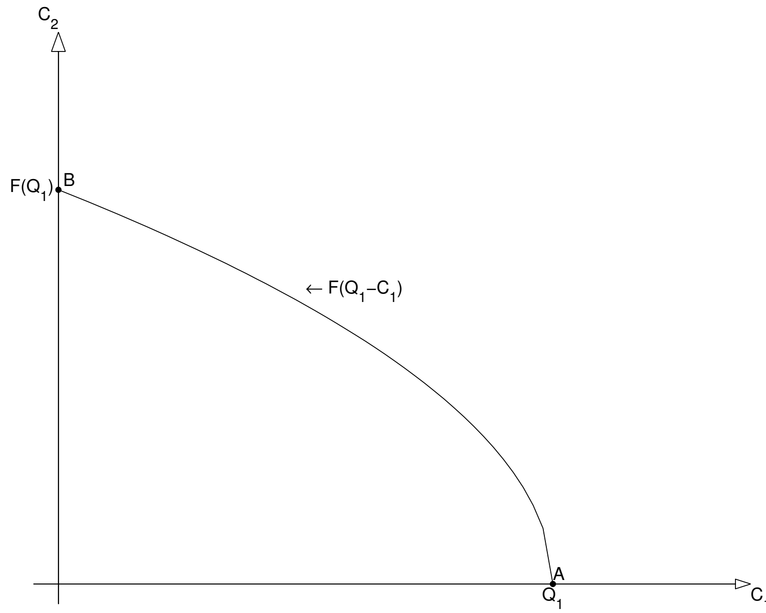
$$Q_1 = C_1 + K_2 - (1-\delta)K_1 \tag{5.9}$$

and

$$Q_2 = C_2 - (1-\delta)K_2 \tag{5.10}$$

The first of these expressions says that output in period 1, Q_1, must be allocated to consumption, C_1, and investment, $K_2 - (1-\delta)K_1$. The second equation has a similar interpretation. Note that because the world ends after period 2, in that period the household chooses to consume the entire undepreciated stock of capital, $(1-\delta)K_2$, so that investment is negative and equal to $-(1-\delta)K_2$. Combining (5.9) and (5.10) and using the fact

Figure 5.4: The production possibility frontier: $C_2 = F(Q_1 - C_1)$



that $Q_2 = F(K_2)$ yields the following equilibrium resource constraint of the economy, also known as the *production possibility frontier* (PPF):

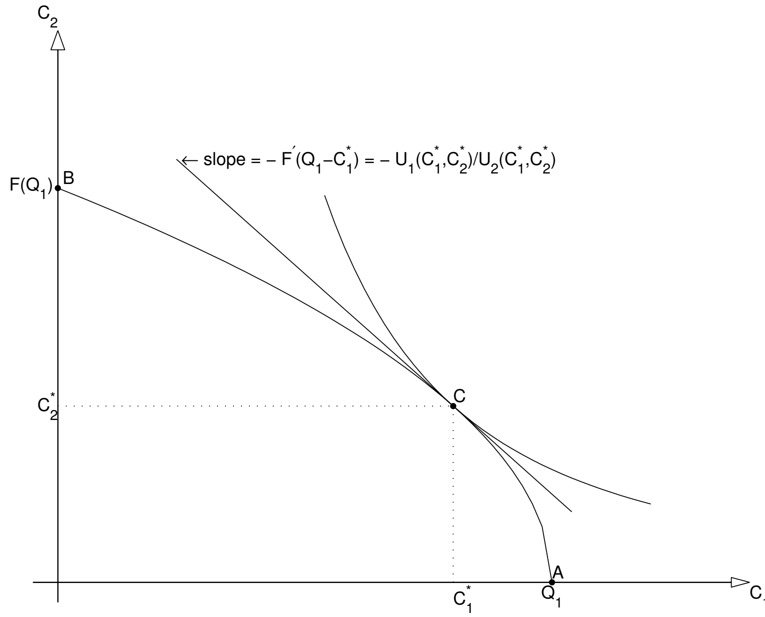
$$C_2 = F(Q_1 + (1-\delta)K_1 - C_1) + (1-\delta)[Q_1 + (1-\delta)K_1 - C_1]$$

The PPF simplifies a great deal when the depreciation rate is assumed to be 100 percent ($\delta = 1$). In this case we have

$$C_2 = F(Q_1 - C_1) \tag{5.11}$$

Figure 5.4 depicts this production possibility frontier in the space (C_1, C_2). Because the production function is increasing and concave, the PPF is downward sloping and concave toward the origin. If in period 1 the household

Figure 5.5: Equilibrium in the model with production: the closed economy case



chooses to carry no capital into the second period by allocating the entire output to consumption ($C_1 = Q_1$), then output in period 2 is nil (point A in the figure). The maximum possible consumption in period 2 can be obtained by setting consumption equal to zero in period 1 ($C_1 = 0$) and using output to accumulate capital (point B in the figure). The slope of the PPF is $-F'(Q_1 - C_1)$.

Which point on the PPF will be chosen in equilibrium, depends on the household's preferences. Figure 5.5 depicts the PPF together with the representative household's indifference curve that is tangent to the PPF. The point of tangency (point C in the figure) represents the equilibrium allocation. At point C, the slope of the indifference curve is equal to the slope of the PPF. From the firm's optimal choice of capital (equation (5.2)) we know that the marginal product of capital, $F'(K_2)$, must equal the rental rate of capital, $r_1 + \delta$. In the special case of a 100 percent depreciation rate, this condition becomes $F'(K_2) = 1 + r_1$. This means that in equilibrium one plus the interest rate is given by (minus) the slope of the PPF at the point of tangency with the household's indifference curve. The important point to note is that in a closed economy the interest rate is determined by domestic factors such as preferences, technologies, and endowments. The interest rate prevailing in the closed economy will in general be different from the world interest rate. Another important point to keep in mind is that in the closed economy savings must always equal investment. To see this, note that in the closed economy savings in period 1, S_1, equals output in period 1 minus consumption in period 1, that is,

$$S_1 = Q_1 - C_1.$$

Recall that investment in period 1 is given by $I_1 = K_2 - (1 - \delta)K_1$. Comparing this expression with (5.9) we have that

$$I_1 = Q_1 - C_1$$

Thus,

$$S_1 = I_1$$

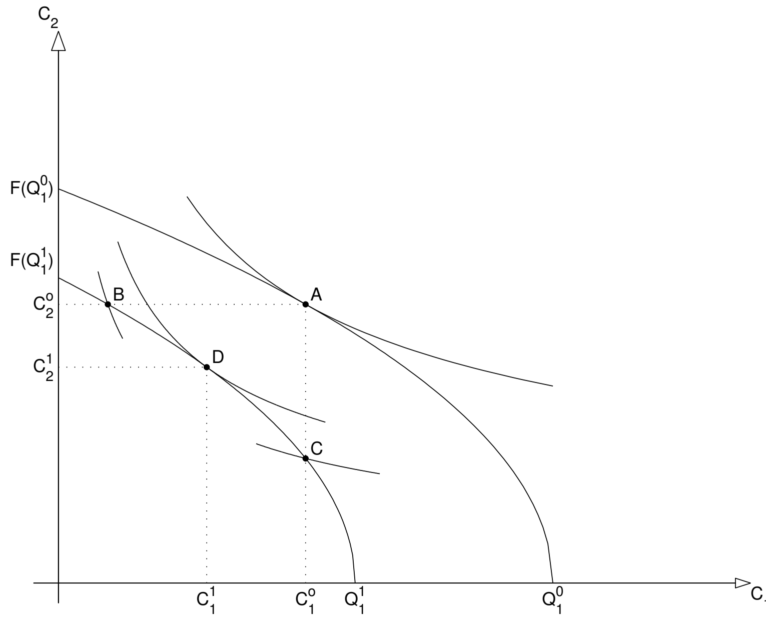
The current account is equal to the difference between savings and investment (see equation (2.9)). Therefore, in a closed economy the current account is always equal to zero. These differences between the open and the closed economies are reflected in the way in which each type of economy adjusts to shocks.

Adjustment to a temporary output shock

Consider a negative transitory shock (such as a natural disaster) that destroys part of output in period 1. In the open economy, households will smooth consumption by borrowing in the international capital market at a constant interest rate, thus running a current account deficit in period 1. In the closed economy, as in the open economy, households desire to borrow against future income in order to smooth consumption. However, in the closed economy, access to international financial markets is precluded. At the same time, the increase in the interest rate has a negative effect on investment in physical capital. The reduction in investment frees up some resources that are used for consumption in period 1 preventing consumption from falling by as much as output.

Figure 5.6 illustrates the adjustment of the closed economy to a decline in output in period 1 from Q_1^0 to $Q_1^1 < Q_1^0$. The economy is initially at point A; consumption in period 1 is C_1^0 and consumption in period 2 is C_2^0. The equilibrium interest rate is given by the slope of the PPF and the indifference curve at point A. It is clear from (5.11) that the decline in output in period 1 produces a parallel shift in the PPF to the left. For example, the distance between points B, on the new PPF, and A, on the old PPF, is equal to the decline in output in period 1, $Q_1^0 - Q_1^1$. Also, at point B, the slope of the new PPF is the same as the slope of the old PPF at point A. Where on the new PPF the equilibrium will be located depends on the shape of the

Figure 5.6: Adjustment to a temporary decline in output in the closed economy



indifference curves. Suppose that at every point on the horizontal segment connecting A and C_2^0, the indifference curves are steeper than at point A. Also, assume that at every point on the vertical segment connecting A with C_1^0 the indifference curves are flatter than at point A. When this property of the indifference curves is satisfied, C_1 and C_2 are said to be normal goods. In addition, because the PPF is strictly concave, as one moves on the PPF from point B to point C, the PPF becomes steeper. Therefore, the indifference curve that crosses point B is, at that point, steeper than the new PPF. Also, the indifference curve that crosses point C is, at that point, flatter than the new PPF. As a result, the new PPF will be tangent to an indifference curve at a point located between points B and C. In the figure, the new equilibrium is given by point D. At the new equilibrium, consumption in period 1 is C_1^1 and consumption in period 2 is C_2^1. Note that consumption in period 1 falls ($C_1^1 < C_1^0$) but by less than the decline in output (the new equilibrium is located to the right of point B). Because output must equal the sum of consumption and investment, the fact that consumption falls by less than output means that investment falls in period 1. At point D, the PPF is steeper than at point B. This means that the negative output shock has induced an increase in the interest rate. Summing up, the effects of a decline in output in period 1 in the closed production economy are: (a) a decline in consumption in period 1 that is less than the decline in output; (b) a decline in savings that is matched by a decline in investment of equal magnitude; and (c) an increase in the interest rate.

We turn next to the analysis of current account determination in a production economy that has access to the world capital market.

5.1.4 Equilibrium in an open economy

In a small open economy households and firms can borrow and lend at an exogenously given world interest rate, which we denote by r^*. Therefore, the interest rate prevailing in the small open economy has to be equal to the world interest rate, that is,

$$r_1 = r^* \tag{5.12}$$

Also, in an open economy, households are not constrained to hold their wealth in the form of domestic capital. In addition to domestic capital, households can hold foreign assets, which are denoted by B^*. Thus,

$$W_0 = K_1 + B_0^* \tag{5.13}$$

and

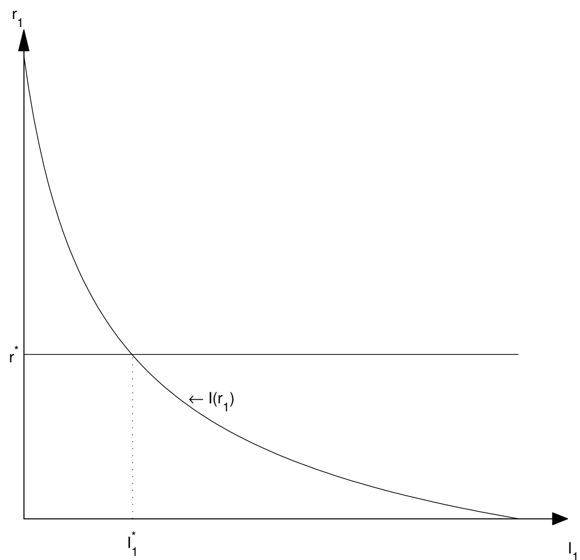
$$W_1 = K_2 + B_1^*.$$

Consider first the optimal investment choice of a domestic firm. Substituting the equilibrium condition $r_1 = r^*$ into equation (5.2) yields the following equilibrium condition determining the capital stock in period 2, which we denote by K_2^*:

$$F'(K_2^*) = r^* + \delta \tag{5.14}$$

This equation implies that the capital stock in period 2 depends only on the world interest rate and the rate of depreciation. Because the marginal product of capital is decreasing in K_2, it follows that K_2^* is a decreasing function of r^*. Recall that investment in period 1 is given by $I_1 = K_2 - (1 - \delta)K_1$. The fact that K_1 is a predetermined variable in period 1 implies that the equilibrium level of investment in period 1, I_1^*, is a decreasing function of r^*. This result marks an important difference between the open and the closed economies. In both economies, investment is a negative function of the interest rate r_1. However, in the closed economy, r_1 depends on preferences and the level of domestic wealth, whereas in the small open economy, r_1 equals r^*, which is independent of domestic preferences and wealth. Figure 5.7 illustrates the determination of investment in period 1 in the small open economy.

Figure 5.7: The equilibrium level of investment, I_1^*



The fact that K_2 is a function of r^* alone implies that the firm's profits in period 2 are also a function of r^* alone. Specifically, using the equilibrium condition $r_1 = r^*$ in (5.1) yields,

$$\Pi_2^* = F(K_2^*) - (r^* + \delta)K_2^*.$$

For simplicity, we assume, as in the case of the closed economy, that $\delta = 1$.[3] Then, profits can be written as,

$$\Pi_2^* = F(K_2^*) - (1 + r^*)K_2^*. \tag{5.15}$$

[3]An implication of assuming that $\delta = 1$ is that $I_1 = K_2$. To see this, recall that $K_2 = (1 - \delta)K_1 + I_1$.

Profits in period 1 are pre-determined and equal to

$$\Pi_1 = Q_1 - (1 + r_0)K_1, \tag{5.16}$$

where $Q_1 \equiv F(K_1)$.

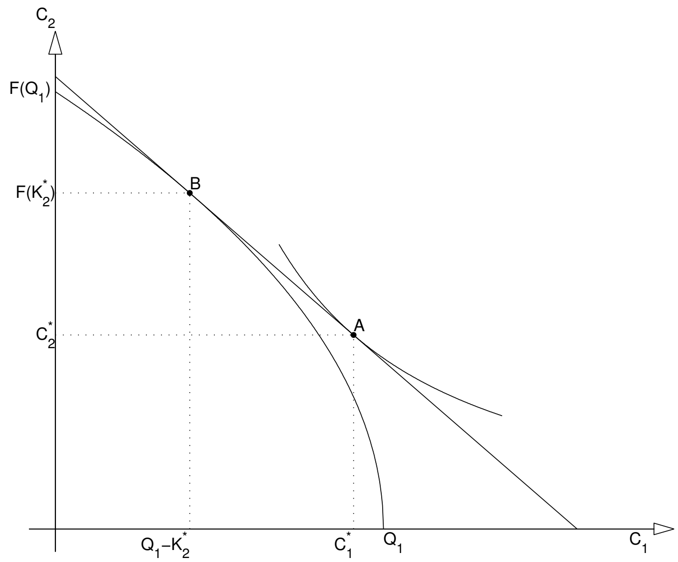
We are now ready to derive the equilibrium resource constraint of the small open production economy. Using the equilibrium conditions (5.12), (5.13), (5.15), and (5.16) to eliminate r_1, W_0, Π_2^*, and Π_1, respectively, from the intertemporal budget constraint of the household, equation (5.8), and assuming for simplicity that $B_0^* = 0$, we get, after rearranging terms,[4]

$$C_2 = (1 + r^*)(Q_1 - K_2^* - C_1) + F(K_2^*)$$

This resource constraint states that in period 2 households can consume whatever they produce in that period, $F(K_2^*)$, plus the amount of foreign assets purchased in period 1 including interest. The amount of foreign assets purchased in period 1 is given by the difference between output in period 1, Q_1, and domestic absorption, $K_2^* + C_1$. The resource constraint describes a linear relationship between C_1 and C_2 with a slope of $-(1 + r^*)$. Figure 5.8 plots this relationship in the plane (C_1, C_2). Clearly, if $Q_1 - K_2^* > 0$, the allocation $C_1 = Q_1 - K_2^*$ and $C_2 = F(K_2^*)$ (point B) is feasible. This allocation corresponds to a situation in which in period 1 the sum of consumption and investment is equal to output, so that the household's net foreign asset holdings in period 1 are exactly equal to zero. This means that point B

[4]The assumption that $B_0^* = 0$ implies that $CA_1 = TB_1$ and $S_1 = Q_1 - C_1$. Can you show why?

Figure 5.8: Equilibrium in the production economy: the small open economy case



would also have been attainable in the closed economy. In other words, point B belongs to the production possibility frontier shown in figure 5.8. As we deduced before, the slope of the PPF is given by $-F'(K_2)$, so that at point B, the slope of the PPF is given by $-F'(K_2^*)$, which, by equation (5.14) equals $-(1+r^*)$.

Note that for any pair (C_1, C_2) lying on the PPF, one can always find another allocation (C_1', C_2') on the resource constraint of the small open economy such that $C_1' \geq C_1$ and $C_2' \geq C_2$. Because the PPF is the resource constraint of the closed economy, it follows that households are better off in the open economy than in the closed economy. We conclude that the imposition of international capital controls (i.e., restrictions to borrowing or lending from the rest of the world) is welfare decreasing in our model.

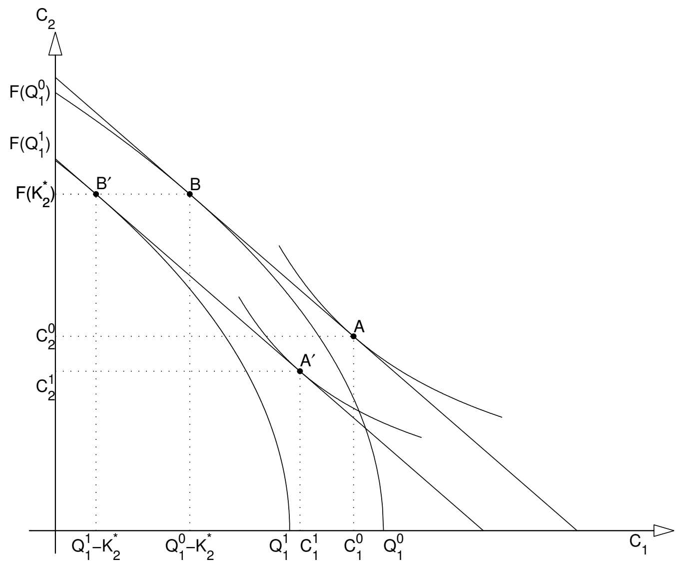
Consumption in each of the two periods is determined by the tangency of the resource constraint with an indifference curve (point A in figure 5.8). In the figure, the trade balance in period 1 is given by minus the distance between C_1^* and $Q_1 - K_2^*$, thus TB_1 is negative. Because B_0^* is assumed to be zero, net investment income in period 1 is zero, which implies that the current account in period 1 is equal to the trade balance in period 1. Saving in period 1, S_1, is given by the distance between Q_1 and C_1^*. Note that in figure 5.8 the current account is in deficit even though saving is positive. This is because investment in physical capital, given by the distance between Q_1 and $Q_1 - K_2^*$, exceeds savings.

5.2 Current account adjustment to output and world-interest-rate shocks

5.2.1 A temporary output shock

Suppose that due to, for example, a negative productivity shock output declines in period 1. Specifically, assume that Q_1^0 falls to Q_1^1. Figure 5.9 describes the situation. Before the shock, consumption in periods 1 and 2 are C_1^0 and C_2^0 (point A), and output in period 2 is $F(K_2^*)$ (point B). When the shock hits the economy, the production possibility frontier shifts to the left in a parallel fashion. Because the economy under consideration is small, the world interest rate, r^*, is unaffected by the temporary output shock, and thus both investment in period 1, $I_1^* = K_2^*$, and output in period 2, $F(K_2^*)$, are unchanged. The slope of the new PPF at $(Q_1^1 - K_2^*, F(K_2^*))$ (point B′) is $-(1 + r^*)$. Because r^* is unchanged, the slope of the new

Figure 5.9: The effect of a temporary output decline in the small-open economy with production



resource constraint continuous to be $-(1+r^*)$. This means that the new resource constraint is tangent to the new production possibility frontier at point B′. If consumption in both periods are normal goods, then C_1 will decline by less than the decline in output (point A′). The fact that C_1 falls by less than Q_1 means that savings in period 1 fall. The current account in period 1 is given by the difference between savings and investment. Since investment is unchanged and savings fall, the current account deteriorates. In period 1, the trade balance equals the current account (recall that $B_0^* = 0$ by assumption). So, the trade balance, like the current account, deteriorates in period 1.

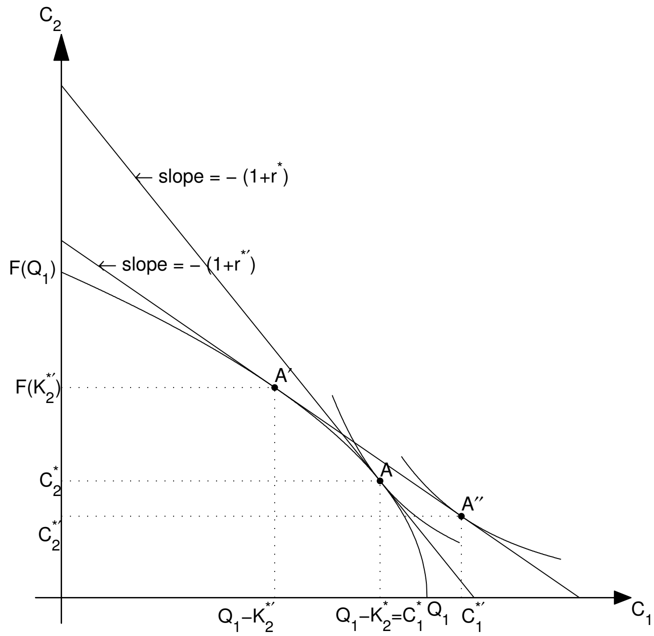
The adjustment in the current account to a temporary output shock in the production economy considered here is qualitatively equivalent to the adjustment in the endowment economy studied in chapter 3. This is because investment is unaffected so that, as in the endowment economy, the response of savings determines the behavior of the current account. The intuition behind this result is the same as in the endowment economy: the output shock is transitory, so agents choose to smooth consumption by borrowing abroad (i.e., by dissaving). The economy enters period 2 with a larger foreign debt, whose repayment requires a trade balance surplus. Because neither investment nor output in period 2 are changed by the output shock, the trade balance surplus must be brought about through a reduction in C_2.

5.2.2 A world-interest-rate shock

Consider now a decrease in the world interest rate from r^* to $r^{*\prime} < r^*$. For simplicity, let us assume that before the shock, the current account balance is zero. This equilibrium is given by point A in figure 5.10. In response to the decline in the interest rate, the resource constraint becomes flatter and is tangent to the PPF at point A′, located northwest of point A. The lower interest rate induces an increase in investment in period 1. Consider next the effect on consumption. By the substitution effect C_1 tends to increase. In addition, households experience a positive income effect originated in the fact that the lower interest rate increases profits in period 2, Π_2. This positive income effect reinforces the substitution effect on C_1. Thus, in period 1 consumption increases (point A″) and savings fall. The fact that investment increases and savings fall implies that the current account and the trade balance deteriorate (recall that $CA_1 = S_1 - I_1$ and that $B_0^* = 0$).

As in the endowment economy, in the production economy the decline

Figure 5.10: A decline in the world interest rate from r^* to $r^{*\prime}$



in the world interest rate generates a deterioration in the trade balance and the current account. However, in the production economy the decline in the current account is likely to be larger because of the increase in investment—an element absent in the endowment economy.

5.3 Exercises

1. **An Economy With Investment**

Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function

$$\ln(C_1) + \ln(C_2),$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively, and ln denotes the natural logarithm. In period 1, the household receives an endowment of $Q_1 = 10$. In period 2, the household receives profits, denoted by Π_2, from the firms it owns. Households and firms have access to financial markets where they can borrow or lend at the interest rate r_1. (r_1 is the interest rate on assets held between periods 1 and 2.)

Firms invest in period 1 to be able to produce goods in period 2. The production technology in period 2 is given by

$$Q_2 = \sqrt{I_1},$$

where Q_2 and I_1 denote, respectively, output in period 2 and investment in period 1.

Assume that there exists free international capital mobility and that the world interest rate, r^*, is 10% per period (i.e., $r^* = 0.1$). Finally, assume that the economy's initial net foreign asset position is zero ($B_0^* = 0$).

(a) Compute the firm's optimal levels of period-1 investment and period-2 profits.

(b) State the maximization problem of the representative household and solve for the optimal levels of consumption in periods 1 and 2.

(c) Find the country's net foreign asset position at the end of period 1, the trade balance in periods 1 and 2, and the current account in periods 1 and 2.

(d) Now consider an investment surge. Specifically, assume that as a result of a technological improvement, the production technology becomes $Q_2 = 2\sqrt{I_1}$. Find the equilibrium levels of savings, investment, the trade balance, the current account, and the country's net foreign asset position in period 1. Compare your results with those obtained in items (a)-(c) providing interpretation and intuition.

2. **Financial Crises, Bailouts, And The Current Account**

Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function

$$\sqrt{C_1 C_2}$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively. In period 1, the household receives an endowment of $Q_1 = 10$. In period 2, the household receives profits, denoted by Π_2, from the firms it owns. In period 1, households and firms have access to financial markets where they can borrow or lend at the interest rate r_1.

Firms borrow in period 1 to invest in physical capital. They are subject to a collateral constraint of the form

$$D_1^f \leq \kappa_1,$$

where D_1^f denotes the amount of debt assumed by the firm in period 1 and κ_1 denotes the value of the firm's collateral. Suppose that κ_1 equals 4. In turn, firms use the physical capital purchased in period 1 to produce final goods in period 2. The production technology in period 2 is given by

$$Q_2 = 6I_1^{1/3},$$

where Q_2 and I_1 denote, respectively, output in period 2 and investment in period 1. Assume that there exists free international capital mobility and that the world interest rate, r^*, is 10% per period. Finally, assume that the economy's initial net foreign asset position is zero ($B_0^* = 0$).

(a) Compute the firm's optimal levels of period-1 investment and period-2 profits. Is the collateral constraint binding in period 1? Explain.

(b) State the maximization problem of the representative household and derive the associated optimality condition.

(c) Solve for the equilibrium levels of period 1 consumption, the country's net foreign asset position (B_1^*), the trade balance, and the current account.

(d) Now suppose that a financial panic causes banks to lower their assessment of the value of firms' collateral. Specifically, suppose that κ_1 falls from 4 to 1. Solve for the equilibrium levels of investment , consumption, the trade balance, the current account, and the country's net asset position in period 1, and output and profits in period 2. Provide intuition.

(e) **A Bailout.** Suppose that as a way to mitigate the financial crisis, in period 1 the government levies a tax on households, denoted T_1, and lends the proceeds to firms at the world interest rate. Let $T_1 = 0.5$, and let D_1^{fG} denote the debt that firms owe to the government and D_1^{fB} the debt that firms owe to private banks. Continue to assume that lending of private banks to firms is limited by the collateral constraint $D_1^{fB} \leq \kappa_1$ and that $\kappa_1 = 1$. In period 2, the government collects loan payments from firms and rebates the whole amount (including interest) to households in the form of a subsidy. State the household's and firm's optimization problems. Compute the equilibrium levels of investment, consumption, the trade balance, the current account, and the country's net foreign asset position in period 1 and output and profits in period 2.

(f) Is the bailout welfare improving? Answer this question by computing the lifetime welfare of the representative household with and without bailout. Discuss your result.

Chapter 6

External Adjustment in Small and Large Economies

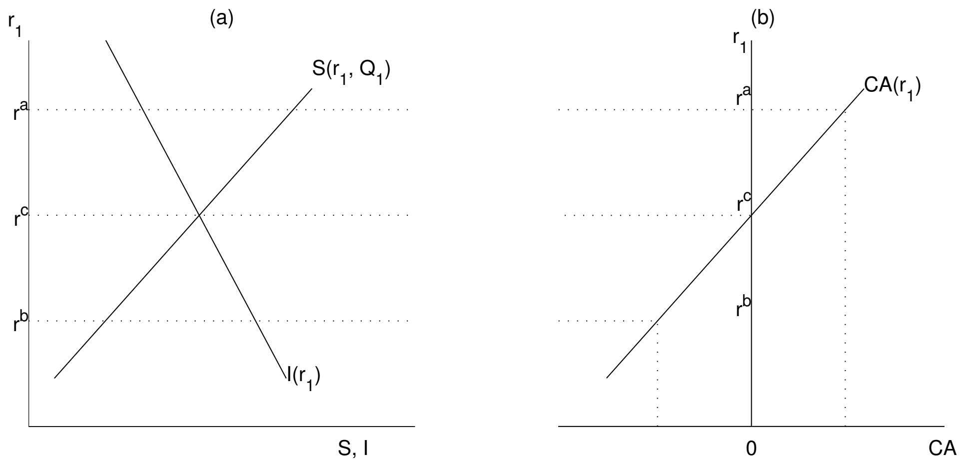
Chapters 3 and 5 provide the microfundations for savings and investment behavior. This chapter takes stock of those results by condensing them in a convenient, user-friendly, synthetic apparatus. The resulting framework provides a simple graphical toolkit to study the determination of savings, investment, and the current account at the aggregate level.

6.1 The Current Account Schedule

Figure 6.1 summarizes the results obtained thus far in chapters 3 and 5. Panel (a) plots the investment and saving schedules.

The investment schedule, $I(r_1)$, is the same as the one shown in figure 5.3. It describes a negative relation between the level of investment and the interest rate resulting from the profit-maximizing investment choice of firms

Figure 6.1: Savings, investment and the current account



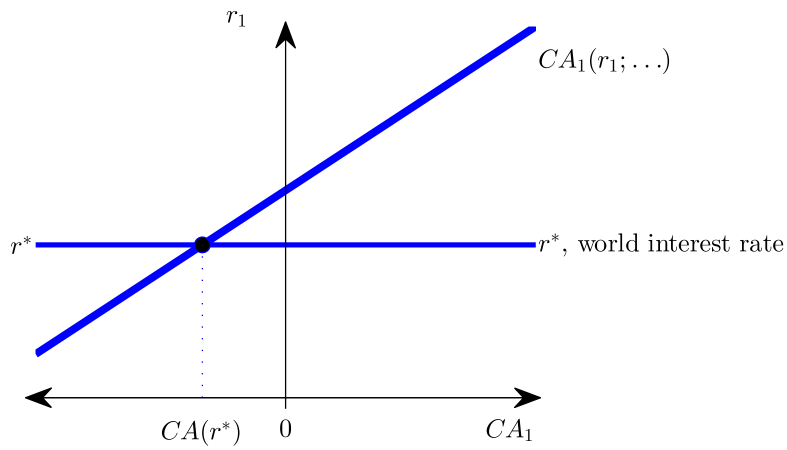
(see equation (5.2)). The schedule is downward sloping because an increase in the interest rate raises the rental cost of capital thus inducing a decline in the demand for equipment, structures, and the like.

The saving schedule, $S(r_1, Q_1)$, relates savings to the interest rate and output in period 1. Savings are increasing in both the interest rate and output. An increase in the interest rate affects savings through three channels: first, it induces an increase in savings as agents substitute future for current consumption. This is called the substitution effect. Second, an increase in the interest rate affects savings through an income effect. If the country is a net foreign debtor, an increase in the interest rate makes its residents poorer and induces them to cut consumption. In this case, the income effect reinforces the substitution effect. However, if the country is a net creditor, then the increase in the interest rate makes households richer, allowing them to consume more and save less. In this case the income effect goes against the substitution effect. Third, an increase in the interest rate has a positive effect on savings because it lowers income from profit in period 2 (Π_2). We will assume that the first and third effects combined are stronger than the second one, so that savings is an increasing function of the interest rate. In section 5.2.1 we analyzed the effects of temporary output shocks in the context of a two-period economy and derived the result that savings are increasing in period 1's output, Q_1. This result arises because an increase in Q_1 represents, holding other things constant, a temporary increase in income, which induces households to increase consumption in *both* periods. Thus, households save more in period 1 in order to consume more in period 2 as well.[1]

Having established the way in which the interest rate and current output affect savings and investment, it is easy to determine the relationship between these two variables and the current account. This is because the current account is given by the difference between savings and investment ($CA_1 = S_1 - I_1$). Panel (b) of figure 6.1 illustrates this relationship. Suppose that the interest rate is r^a. Then savings exceed investment, which implies that the current account is in surplus. If the interest rate is equal to r^c, then investment equals savings and the current account is zero. Note that r_c is the interest rate that would prevail in a closed economy, that is, in an economy that does not have access to international capital markets. For interest rates below r^c, such as r^b, investment is larger than savings so that the country runs a current account deficit. Therefore, as shown in panel (b), the current account is an increasing function of the interest rate. With the

[1] In general, the savings schedule also depends (positively) on initial net foreign asset holdings, B_0^*, and net investment income, $r_0 B_0^*$. Therefore, strictly speaking, the schedule $S(r_1, Q_1)$ embodies the implicit assumption that $B_0^* = 0$.

Figure 6.2: Current Account Determination in a Small Open Economy



help of this graphical apparatus, it is now straightforward to analyze the effects of various shocks on investment, savings, and the current account.

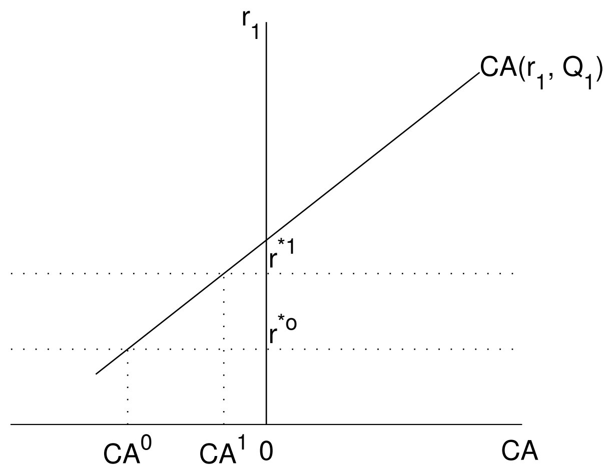
6.2 External Adjustment in a Small Open Economy

In a small open economy with free capital mobility, in equilibrium the domestic interest rate must equal the world interest rate, r^*, that is,

$$r_1 = r^*.$$

Thus we can find the equilbrium level of the current account by simply evaluating the current account schedule at $r_1 = r^*$. Figure 6.2 shows the equilibrium level of the current account, $CA(r^*)$. At point A, the current

Figure 6.3: Current account adjustment to an increase in the world interest rate

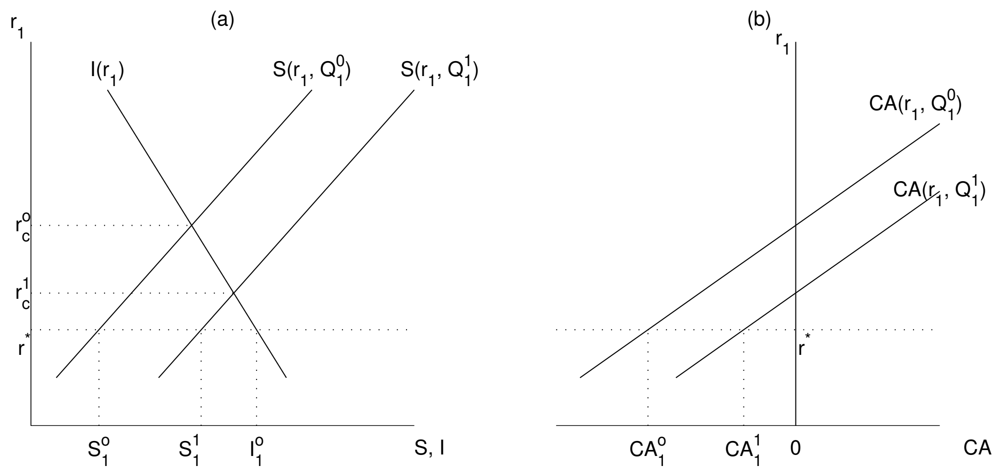


account schedule $CA(r_1; \ldots)$ and the world interest rate intersect.

6.2.1 Interest Rate Shocks

We begin by revisiting the effects of world interest rate shocks. Suppose a small open economy that initially faces the world interest rate r^{*o} as shown in figure 6.3. At that interest rate, the country runs a current account deficit equal to CA^0. Suppose now that the world interest rate rises to $r^{*1} > r^{*o}$. The change in the world interest rate does not shift the current account schedule. Hence the equilibrium value of the current account is given by the point where the (unchanged) current account schedule intersects the new higher world interest rate level. The higher world interest rate encourages domestic saving and forces firms to reduce investment in physical capital. As a result, in equilibrium the current account deficit declines from CA^0 to CA^1.

Figure 6.4: Current account adjustment to a temporary increase in output



6.2.2 Temporary Output Shock

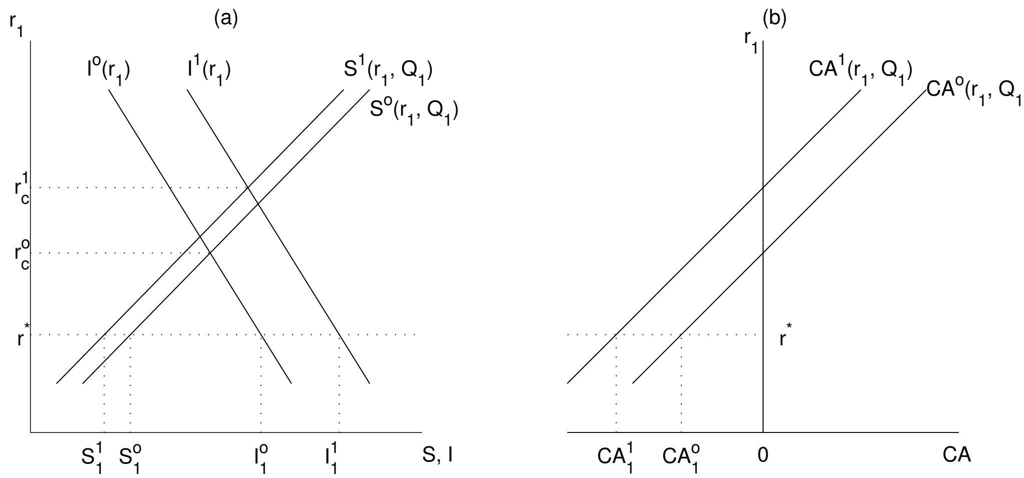
Consider next the effects of a temporary positive income shock, that is, an increase in Q_1. We illustrate the effects of this shock in figure 6.4. Suppose that Q_1 is initially equal to Q_1^0. At the world interest rate r^*, savings are equal to S_1^0, investment is equal to I_1^0, and the current account is $CA_1^0 = S_1^0 - I_1^0$. Suppose now that Q_1 increases to $Q_1^1 > Q_1^0$. This increase in Q_1 shifts the saving schedule to the right because households, in an effort to smooth consumption over time, save part of the increase in income. On the other hand, the investment schedule does not move because investment is not affected by current income. The rightward shift in the savings schedule implies that at any given interest rate the difference between savings and investment is larger than before the increase in income. As a result, the current account schedule shifts to the right. Given the world interest rate, the current account increases from CA_1^0 to CA_1^1. Thus, a temporary increase in income produces an increase in savings, and an improvement in the current account balance while leaving investment unchanged.

Note that if the economy was closed, the current account would be zero before and after the income shock, and the interest rate would fall from r_c^0 to r_c^1. This decline in the interest rate would induce an expansion in investment. Because in the closed economy savings are always equal to investment, savings would also increase.

6.2.3 An investment surge

Suppose that in period 1 agents learn that in period 2 the productivity of capital will increase. For example, suppose that the production function in period 2 was initially given by $F(K_2) = \sqrt{K_2}$ and that due to a technological advancement it changes to $\tilde{F}(K_2) = 2\sqrt{K_2}$. Another example of an investment surge is given by an expected increase in the price of exports. In Norway, for instance, the oil price increase of 1973 unleashed an investment boom of around 10% of GDP. In response to this news, firms will choose to increase investment in period 1 for any given level of the interest rate. This scenario is illustrated in figure 6.5. Initially, the investment schedule is $I^0(r_1)$ and the saving schedule is $S^0(r_1, Q_1)$. Given the world interest rate r^*, investment is I_1^0 and savings is S_1^0. As shown in panel (b), the current account schedule is $CA^0(r_1, Q_1)$, and the equilibrium current account balance is CA_1^0. The news of the future productivity increase shifts the investment schedule to the right to $I^1(r_1)$, and the new equilibrium level of investment is I_1^1, which is higher than I_1^0. The expected increase in productivity might also affect current saving through its effect on expected future

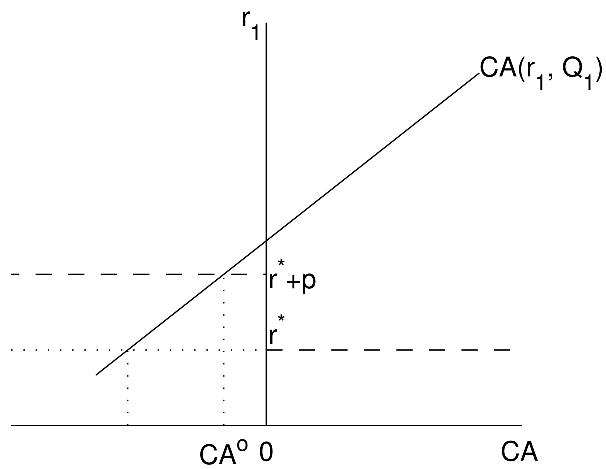
Figure 6.5: An investment surge



income. Specifically, in period 2, firms will generate higher profits which represent a positive income effect for households who are the owners of such firms. Households will take advantage of the expected increase in profits by increasing consumption in period 1, thus cutting savings. Therefore, the savings schedule shifts to the left to $S^1(r_1, Q_1)$ and the equilibrium level of savings falls from S_1^0 to S_1^1. With this shifts in the investment and savings schedules it follows that, for any given interest rate, the current account is lower. That is, the current account schedule shifts to the left to $CA^1(r_1, Q_1)$. Given the world interest rate r^*, the current account deteriorates from CA_1^0 to CA_1^1.

Note that if the economy was closed, the investment surge would trigger a rise in the domestic interest rate from r_c^0 to r_c^1 and thus investment would increase by less than in the open economy.

Figure 6.6: Current account determination in the presence of a constant risk premium

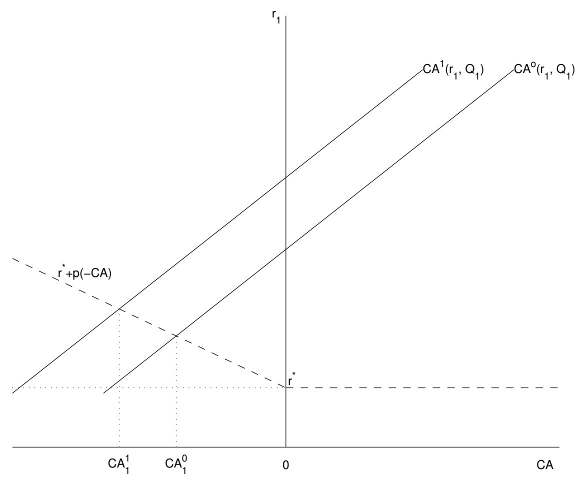


6.3 Country Risk Premia

In practice, the interest rate that emerging countries face on their international loans is larger than the one developed countries charge to each other. This interest rate differential is called the country risk premium, and we denote it by p. Figure 6.6 illustrates the situation of a small open economy facing a country risk premium. In the graph it is assumed that the premium is charged only when the country is a debtor to the rest of the world. Suppose that the initial asset position, B_0^*, is zero. In this case, the country is a debtor at the end of period one if it runs a current account deficit in period one and a creditor if it runs a current account surplus in period one. Furthermore, the stock of debt at the end of period one is equal to the current account deficit in period 1, that is, in this case $B_1^* = -CA_1$. It follows that the country risk premium applies whenever the current account is in deficit. Thus, the interest rate faced by the small open economy is r^* when $CA > 0$ and $r^* + p > r^*$ when $CA < 0$. In figure 6.6, given the world interest rate r^* and the country risk premium p, the country runs a current account deficit equal to CA^0. Note that the current account deficit is smaller than the one that would obtain if the country faced no risk premium. Thus, if the current account is negative, an increase in the risk premium reduces the current account deficit in exactly the same way as an increase in the interest rate.

A more realistic specification for the interest rate faced by developing countries is one in which the country risk premium is an increasing function of the country's net foreign debt. Given our assumption that the initial net foreign asset position is zero, the country's foreign debt at the end of period 1 is given by its current account deficit. Thus, we can represent the country risk premium as an increasing function of the current account deficit, $p(-CA)$ (see figure 6.7). Consider now the response of the current account to an investment surge like the one discussed in section 6.2.3. In response to the positive investment shock, the current account schedule shifts to the left from $CA^0(r_1, Q_1)$ to $CA^1(r_1, Q_1)$. As a result, the current account deteriorates from CA_1^0 to CA_1^1 and the interest rate at which the country can borrow internationally increases from $r^* + p(-CA_1^0)$ to $r^* + p(-CA_1^1)$. The resulting deterioration in the current account is, however, smaller than the one that would have taken place had the country risk premium remained constant.

Figure 6.7: Current account determination in the presence of an increasing risk premium

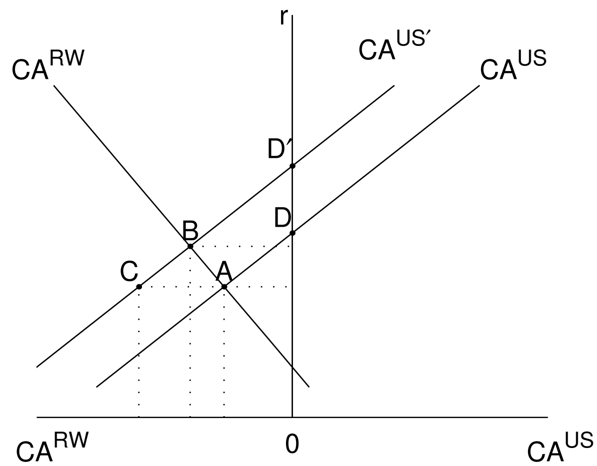


6.4 External Adjustment in a Large Open Economy

Thus far, we have considered current account determination in a small open economy. We now turn to the determination of the current account in a large open economy like the United States. Let's divide the world into two regions, the United States (US) and the rest of the world (RW). Because a U.S. current account deficit represents the current account surplus of the rest of the world and conversely, a U.S. current account surplus is a current account deficit of the rest of the world, it follows that the world current account must always be equal to zero; that is,

$$CA^{US} + CA^{RW} = 0,$$

Figure 6.8: Current account determination in a large open economy



where CA^{US} and CA^{RW} denote, respectively, the current account balances of the United States and the rest of the world.

Figure 6.8 shows the current account schedules of the U.S. and the rest of the world. The innovation in the graph is that the current account of the rest of the world is measured from right to left, so that to the left of 0 the rest of the world has a CA surplus and the U.S. a CA deficit, whereas to the right of 0, the U.S. runs a CA surplus and the rest of the world a CA deficit. Equilibrium in the world capital markets is given by the intersection of the CA^{US} and CA^{RW} schedules. In the figure, the equilibrium is given by point A, at which the U.S. runs a current account deficit and the rest of the world a current account surplus.

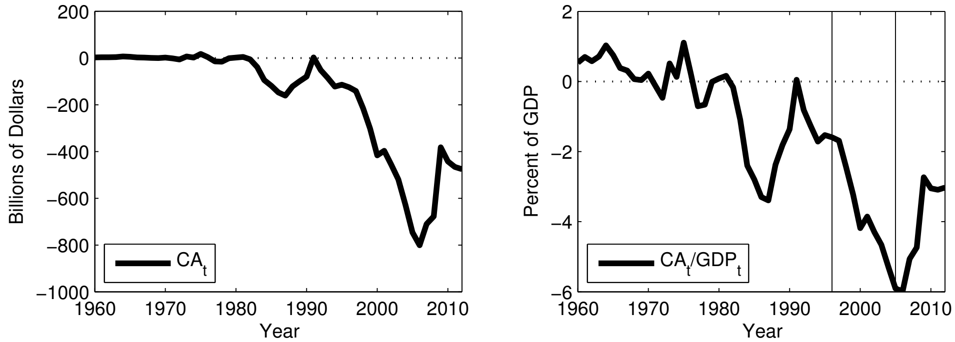
Consider now an investment surge in the U.S. that shifts the CA^{US} schedule to the left to $CA^{US'}$. The new equilibrium is given by point B, where the schedule $CA^{US'}$ and the schedule CA^{RW} intersect. At point B, the world interest rate is higher, the US runs a larger CA deficit, and the rest of the world runs a larger CA surplus. Note that because the U.S. is a large open economy, the investment surge produces a large increase in the demand for loans, which drives world interest rates up. As a result, the deterioration in the U.S. current account is not as pronounced as the one that would have resulted if the interest rate had remained unchanged (point C in the figure). Note further that the increase in the U.S. interest rate is smaller than the one that would have occurred if the US economy was closed (given by the distance between D′ and D).

6.5 The Global Saving Glut Hypothesis

Between 1995 and 2005, the U.S. current account deficit experienced a dramatic increase from \$125 to \$ 623 billion dollars. This \$500 billion dollar increase brought the deficit from a relatively modest level of 1.5 percent of GDP in 1995 to close to 6 percent of GDP in 2005. With the onset of the great recession of 2007, the ballooning of the current account deficits came to an abrupt stop. By 2009, the current account deficit had shrunk back to 3 percent of GDP. (See figure 6.9.)

An important question is what factors are responsible for these large swings in the U.S. current account. In particular, we wish to know whether the recent rise and fall in the current account deficit were driven by domestic or external factors.

Figure 6.9: The U.S. Current Account Balance: 1960-2012



Data Source: BEA. The vertical lines indicate the years 1996 and 2005.

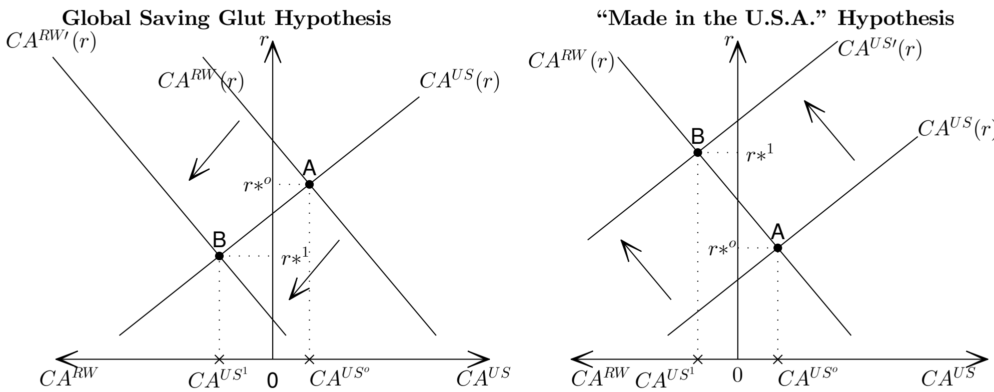
6.5.1 The Period 1996 to 2006

In 2005 Ben Bernanke, then a governor of the Federal Reserve, gave a speech in which he argued that the deterioration in the U.S. current account deficits between 1996 and 2004 were caused by external factors.[2] He coined the term 'global saving glut' to refer to these external factors. In particular, Bernanke argued that the rest of the world experienced a heightened desire to save but did not have incentives to increase domestic capital formation in a commensurate way. As a result, the current account surpluses of the rest of the world had to be absorbed by current account deficits in the United States.

Much of the increase in the desired current account surpluses in the rest of the world during this period originated in higher desired savings in emerging market economies. In particular, Bernanke attributes the increase in the desire to save to two factors: (1) Increased foreign reserve accumulation to

[2]Bernanke, Ben S., "The Global Saving Glut and the U.S. Current Account Deficit," Homer Jones Lecture, St. Louis, Missouri, April 14, 2005.

Figure 6.10: U.S. Current Account Deterioration: Global Saving Glut or "Made in the U.S.A."



avoid or be better prepared to face future external crises of the type that had afflicted emerging countries in the 1990s. And (2) Currency depreciations aimed at promoting export-led growth.

The global saving glut hypothesis was unconventional at the time. The more standard view was that the large U.S. current account deficits were the results of economic developments inside the United States and unrelated to external factors. Bernanke refers to this alternative hypothesis as the "Made in the U.S.A." view.

How can we tell which view is right, the global saving glut hypothesis or the "Made in the U.S.A." hypothesis? To address this question, we can use the graphical tools developed in section 6.4 of this chapter. The left panel of figure 6.10 illustrates the effect of a desired increase in savings in the rest of the world. The initial position of the economy, point A, is at the intersection of the CA^{US} and CA^{RW} schedules. In the initial equilibrium, the U.S. current account equals CA^{US^0} and the world interest rate equals

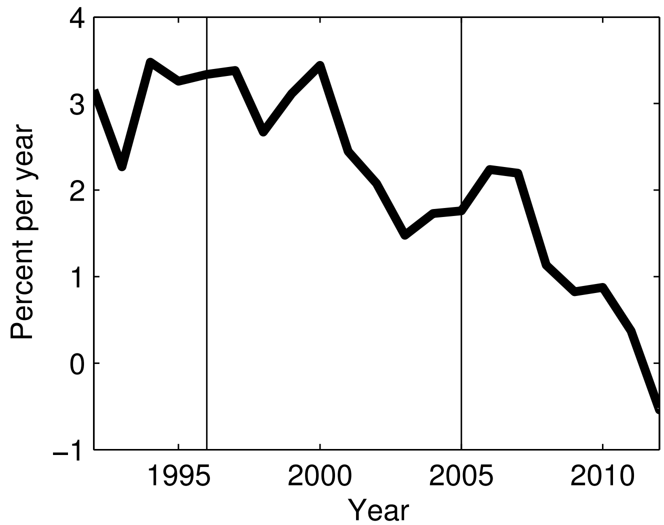
r^{*0}. The increase in the desired savings of the rest of the world shifts the current account schedule of the rest of the world down and to the left as depicted by the schedule $CA^{RW\prime}$. The new equilibrium, point B, features a deterioration in the current account deficit of the U.S. from CA^{US^0} to CA^{US^1} and a fall in the world interest rate from r^{*^0} to r^{*^1}. Intuitively, the United States will borrow more from the rest of the world only if it becomes cheaper to do so, that is, only if the interest rate falls. This prediction of the model implies that if the global saving glut hypothesis is valid, then we should have observed a decline in the interest rate.

The "Made in the U.S.A." hypothesis is illustrated in the right hand panel of figure 6.10. Again, in the initial equilibrium, point A, the U.S. current account equals CA^{US^0} and the world interest rate equals r^{*^0}. Under this view, the current account schedule of the rest of the world is unchanged and instead the current account schedule of the United States shifts to the left as depicted by the schedule $CA^{US\prime}$. The new equilibrium, point B, features a deterioration in the current account deficit of the U.S. from CA^{US^0} to CA^{US^1} and a rise in the world interest rate form $r^{*^0}$to $r^{*^1} > r^{*0}$. Both hypotheses can explain a deterioration in the U.S. current account. However, the global saving glut hypothesis implies that the CA deterioration should have been accompanied by a decline in world interest rates, whereas the "Made in the U.S.A." hypothesis implies that world interest rates should have gone up. Hence we can use data on the behavior of interest rates to find out which hypothesis is right.

Figure 6.11 plots the world interest rate.[3] It shows that over the period

[3]The world interest rate is computed as the difference between the 10-year constant

Figure 6.11: The World Interest Rate: 1992-2012



Note. The world interest rate is approximated by the difference between the rate on 10-year U.S. Treasury securities and expected inflation. The vertical lines indicate the years 1996 and 2005, respectively.

in question, 1996 to 2005, interest rates fell, validating the global saving glut hypothesis and rejecting the "Made in the U.S.A." hypothesis.

6.5.2 The Period 2006 to 2012

Can the global saving glut hypothesis also explain changes in U.S. current account dynamics after 2005? Figure 6.9 shows that at its peak in 2006 the U.S. current account deficit had reached 6 percent of GDP. Over the following 3 years, the deficit was reduced to half, or 3 percent of GDP. Under the global saving glut hypothesis, this reduction in the current account deficit would be attributed to a decline in desired savings in the rest of the world.

maturity Treasury rate and expected inflation. Expected inflation in turn is measured as the median CPI-inflation forecast over the next 10 years and is taken from the Survey of Professional Forecasters.

Again we can use the graphical tools developed earlier in this chapter to evaluate the plausibility of this view. Consider the left panel of figure 6.10. Assume that the initial equilibrium is at point B, where the world interest rate is equal to r^{*1} and the U.S. current account deficit is equal to CA^{US^1}. We can represent a decline in desired savings in the rest of the world as a shift up and to the right in the current account schedule of the rest of the world. For simplicity, assume that this adjustment is shown as a return of the current account schedule of the rest of the world back to its original position given by CA^{RW} so that the new equilibrium is given by point A. This shift in the current account schedule of the rest of the world causes the U.S. current account to improve from CA^{US^1} to CA^{US^0} and the interest rate to rise from r^{*1} to r^{*^0}. It follows that under the global saving glut hypothesis, the V-shape of the U.S. current account balance observed between 1996 and 2009 (see figure 6.9), should have been accompanied by a V-shaped pattern of the interest rate. However, figure 6.11 shows that the interest rate does not display a V-shaped pattern as predicted by the global saving glut hypothesis. In fact, since 2005 the interest rate has declined further rejecting the global saving glut hypothesis as an explanation of U.S. current account dynamics since 2005.

We conclude that the global saving glut hypothesis presents a plausible explanation for the observed developments in the U.S. current account deficit over the period 1996-2005. At the same time, the empirical evidence, in particular, the behavior of interest rates, suggests that the dynamics of the U.S. current account since 2005 were not primarily driven by external factors, but instead by domestic disturbances.

6.6 Optimal Capital Controls in a Two-Country Model

In this section, we study optimal capital controls in a two-country model. In such a model each country is a big player. As a result when one of the two countries changes its desired net foreign asset position, the world supply of assets may change significantly, and hence the world interest rate will in general be affected. Because both countries are big, each of them individually has market power in the international financial market. That is, each country has an incentive to act like a monopolist and to chose a world interest rate that is different from the one that would arise if it did not have market power.

The reason why deviations from free capital mobility might be welfare improving from the perspective of one of the countries, is that when countries internalize the effect that their choices regarding net foreign assets holdings have on world interest rates, we say they act strategically. Specifically, a country that is borrowing has incentives to improve its current account so as to depress the world interest rate, thereby lowering debt service costs.

Consider a two-period model with two large open endowment economies, US for the United States and C for China, and a single traded good. In both countries household preferences over period-1 consumption, C_1, and period-2 consumption, C_2, are given by the following time-separable utility function

$$U(C_1, C_2) = \ln C_1 + \ln C_2.$$

The endowment in country US is constant over time and equal to Q, that is,

$$Q_1^{US} = Q$$

and

$$Q_2^{US} = Q.$$

By contrast, the endowment in country C is growing over time. Let Q_1^C denote the endowment in country C in period 1, and Q_2^C its endowment in period 2. Assume that

$$Q_1^C = \frac{Q}{2}$$

and

$$Q_2^C = Q.$$

Further assume that the net foreign asset position at the beginning of period 1 is zero in both countries.

The Current Account Schedule of country U.S., $CA_1^{US}(r)$

The problem of households in country US consist in choosing consumption in period 1, C_1^{US}, consumption in period 2, C_2^{US}, and net foreign assets at the end of period 1, B_1^{US} so as to maximize lifetime utility, which is given by:

$$U(C_1^{US}, C_2^{US}) = \ln C_1^{US} + \ln C_2^{US}. \tag{6.1}$$

The budget constraint of households in country US in period 1 is given by:

$$C_1^{US} + B_1^{US} = Q_1^{US} \tag{6.2}$$

and in period 2, it is given by

$$C_2^{US} = Q_2^{US} + (1 + r_1)B_1^{US}. \tag{6.3}$$

In stating the budget constraints, we used the fact that in any equilibrium it must be the case that at the end of period 2 each country must have a zero net foreign asset position, that is, we imposed that $B_2^{US} = 0$. Households take their endowments and the domestic interest rate on assets held from period 1 to period 2, r_1, as exogenously given.

Use the budget constraint in period 1 to eliminate B_1^{US} from the period-2 budget constraint, to obtain the following present value budget constraint:

$$C_1^{US} + \frac{C_2^{US}}{1 + r_1} = Q_1^{US} + \frac{Q_2^{US}}{1 + r_1}. \tag{6.4}$$

Solve this present value budget constraint for C_1^{US} and use it to replace C_1^{US} in the utility function, equation (6.1). This yields:

$$\ln\left[-\frac{C_2^{US}}{1 + r_1} + Q_1^{US} + \frac{Q_2^{US}}{1 + r_1}\right] + \ln C_2^{US}.$$

Now take the first-order condition with respect to C_2^{US} to obtain:

$$\frac{1}{C_1^{US}}(-1)\frac{1}{1 + r_1} + \frac{1}{C_2^{US}} = 0.$$

Rearranging this expression we find that at the optimum:

$$C_1^{US} = \frac{C_2^{US}}{1+r_1}.$$

The interpretation of this first-order condition is that at the optimum the slope of the indifference curve, given by C_2^{US}/C_1^{US}, must be the same as the slope of the intertemporal budget constraint, given by $1+r_1$. Next use the intertemporal budget constraint, equation (6.4), to find the optimal level of period 1 consumption as a function of the interest rate:

$$\begin{aligned} C_1^{US} + \frac{C_2^{US}}{1+r_1} &= Q_1^{US} + \frac{Q_2^{US}}{1+r_1} \\ C_1^{US} + C_1^{US} &= Q + \frac{Q}{1+r_1} \\ C_1^{US} &= \frac{1}{2}\left(Q + \frac{Q}{1+r_1}\right) \end{aligned}$$

From here we find that in equilibrium it must be the case that:

$$CA_1^{US} = B_1^{US} - B_0^{US} = B_1^{US} - 0 = Q - C_1^{US} = \frac{1}{2}Q - \frac{1}{2}\frac{Q}{1+r_1}$$

We can simplify this expression to obtain the **current account schedule of country US**

$$CA_1^{US}(r) = \frac{1}{2}Q\frac{r_1}{1+r_1}. \tag{6.5}$$

Notice that the current account schedule is upward sloping in the space (CA, r), that is, the higher the interest rate, the higher the current account.

The Current Account Schedule of country C, $CA_1^C(r)$

Next we wish to find country C's current account schedule, which we denote $CA^C(r_1^c)$, where r_1^c is the domestic interest rate in country C for assets held from period 1 to period 2.

Households in country C solve the same maximization problem as households in country US but for the fact that country C's period-1 endowment and domestic interest rate are potentially different. Formally, the problem of the representative household in country C is given by

$$\max_{C_1^C, C_2^C, B_1^C} \ln C_1^C + \ln C_2^C$$

subject to

$$C_1^C + B_1^C = Q_1^C \tag{6.6}$$

and

$$C_2^c = Q_2^c + (1 + r_1^c)B_1^c, \tag{6.7}$$

where again we imposed that net foreign assets at the end of period 2 will be zero.

The first-order conditions associated with this problem are:

$$\frac{C_2^C}{C_1^C} = 1 + r_1^c \tag{6.8}$$

and

$$C_1^C + \frac{C_2^C}{1 + r_1^c} = Q_1^C + \frac{Q_2^C}{1 + r_1^c} \tag{6.9}$$

Using the facts that $Q_1^C = Q/2$ and $Q_2^C = Q$, and following the same steps as above, we find that period-1 consumption in equilibrium is given by the following function of r_1^c:

$$C_1^C = \frac{1}{2}\left(\frac{Q}{2} + \frac{Q}{1+r_1^c}\right)$$

And the current account schedule of country C is given by

$$CA_1^C = B_1^C - B_0^C = B_1^C - 0 = \frac{Q}{2} - C_1^C$$

We can simplify this expression to obtain the current account schedule of country C as a function of the interest rate,

$$CA_1^C(r_1^c) = \frac{Q}{4} - \frac{1}{2}\frac{Q}{1+r_1^c}. \tag{6.10}$$

It follows from this expression that, unless the interest rate exceeds 100 percent, i.e., $r_1^c > 1$, country C will run a current account deficit in period 1. That is, to induce country C to run a current account surplus in period 1, the interest rate must be higher than 100 percent. When country C runs a current account deficit in period 1, it is borrowing against its period-2 income. Country C has much higher income in period 2 than in period 1. To smooth consumption over time country C thus has to borrow in period 1 and repay in period 2. Notice that therefore the example given here is one in which the country that is expecting endowment growth is the country that will run a current account deficit and the country that is not expecting any sizeable change in its endowment in the future will run a current account surplus. Thus, this example cannot explain why the country that is expected to grow faster runs a surplus (China) against the slower growing country (the U.S.).

6.6.1 Market Clearing in World Capital Markets

In equilibrium the world current account balance has to be zero, that is,

$$CA_1^C + CA_1^{US} = 0. \tag{6.11}$$

Can you verify that this equation implies that in equilibrium the world endowment of goods is equal to world wide absorption, that is,

$$C_1^{US} + C_1^C = \frac{3}{2}Q$$

and

$$C_2^{US} + C_2^C = 2Q.$$

6.6.2 Equilibrium Under Free Capital Mobility

Consider first the case that there is free capital mobility in both countries. Then in equilibrium the interest rate must be the same in both countries, that is,

$$r_1 = r_1^c. \tag{6.12}$$

In this case the world interest rate is determined as the solution to (6.11). Combining (6.10), (6.5), (6.12), and (6.11) yields:

$$\frac{1}{2}Q\frac{r_1}{1+r_1} + \frac{Q}{4} - \frac{1}{2}\frac{Q}{1+r_1} = 0$$

Solving for r_1, we obtain

$$r_1 = \frac{1}{3}.$$

It follows that under free capital mobility the world interest rate is 33 percent. Using this information, we can find that

$$C_1^{US} = \frac{1}{2}\left(Q + \frac{Q}{1+1/3}\right) = \frac{7}{8}Q$$

$$CA_1^{US} = Q - C_1^{US} = \frac{1}{8}Q$$

Country US consumes $\frac{7}{8}$ of its endowment and saves the rest. Hence it runs a current account surplus of $\frac{1}{8}Q$. It follows that the current account deficit of country C must be equal to $-\frac{1}{8}Q$, that is,

$$CA_1^C = -\frac{1}{8}Q.$$

And thus consumption in country C is given by

$$C_1^C = \frac{Q}{2} - CA_1^c = \frac{5}{8}Q.$$

In period 2, country US consumes

$$C_2^{US} = Q_2^{US} + (1+r_1)B_1^{US} = Q + \frac{4}{3}\frac{1}{8}Q = \frac{7}{6}Q.$$

And country C consumes

$$C_2^C = Q_2^c + (1+r_1)B_1^c = Q - \frac{4}{3}\frac{1}{8}Q = \frac{5}{6}Q.$$

The level of welfare under free capital mobility can be found by evaluating the utility function at the respective equilibrium consumption levels:

$$U(C_1^{US}, C_2^{US}) = \ln C_1^{US} + \ln C_2^{US} = \ln \frac{7}{8}Q + \ln \frac{7}{6}Q = \ln\left(\frac{49}{48}Q^2\right) = \ln\left(1.0208Q^2\right) \tag{6.13}$$

and welfare in country C is equal to

$$U(C_1^c, C_2^c) = \ln C_1^c + \ln C_2^c = \ln \frac{5}{8}Q + \ln \frac{5}{6}Q = \ln\left(\frac{25}{48}Q^2\right) = \ln\left(0.5208Q^2\right)$$

Can you show that both countries are better off under free capital mobility than under financial autarky?

6.6.3 Equilibrium when country C imposes capital controls

Now suppose country C behaves strategically and manipulates capital flows (by means of capital controls) to obtain a value of the world interest rate, r_1, that maximizes welfare of agents in country C. Given that under free capital mobility country C is a borrower, if current account manipulation results in a lower world interest rate, then this in effect is a positive income effect for country C. So our conjecture is that under optimal current account manipulation the world interest rate will be lower.

We will continue to assume that there is free capital mobility in country US. Specifically, the demand for international funds by country US is still given by:

$$CA^{US}(r_1) = \frac{1}{2}Q\frac{r_1}{1+r_1}$$

How can we model strategic behavior, or optimal capital controls, in country C? We assume that the government of country C picks the interest rate so as to maximize the welfare of its agents taking into account, or internalizing, the effect that its own demand for funds has on the world interest rate. Specifically, the government of country C knows that the world capital market must clear

$$B_1^C + B_1^{US} = 0$$

and that for any given interest rate the international bond demand of country US is

$$B^{US}(r_1) = \frac{1}{2} Q \frac{r_1}{1+r_1}.$$

Combining these two expressions we obtain:

$$B_1^c = -\frac{1}{2} Q \frac{r_1}{1+r_1}. \tag{6.14}$$

The government of country C also knows that its budget constraints in periods 1 and 2 are:

$$C_1^C + B_1^C = \frac{1}{2} Q$$

$$C_2^C = Q + (1+r_1) B_1^C.$$

Using (6.14) to eliminate B_1^c from these two budget constraints yields:

$$C_1^C = \frac{1}{2} Q \frac{1+2r_1}{1+r_1} \tag{6.15}$$

and

$$C_2^C = Q - \frac{1}{2}r_1 Q = \frac{1}{2}Q[2 - r_1] \tag{6.16}$$

It follows that period 1 and period 2 consumption in country C, C_1^C and C_2^C, are only functions of the world interest rate r_1, and hence that the lifetime utility of agents in country C can be written as a function of the world interest rate only. Use the above two expressions to eliminate C_1^C and C_2^C from the utility function of the representative agent in country C to obtain:

$$\begin{aligned} V(r_1) = U(C_1^C(r_1), C_2^C(r_1)) &= \ln C_1^C(r_1) + \ln C_2^C(r_1) \\ &= \ln\left(\frac{1}{2}Q\frac{1+2r_1}{1+r_1}\right) + \ln\left(\frac{1}{2}Q[2-r_1]\right) \\ &= \ln\left(\frac{1}{4}Q^2\right) + \ln(1+2r_1) - \ln(1+r_1) + \ln(2-r_1) \end{aligned}$$

The function $V(r_1)$ is known as country C's indirect utility function. Now pick the world interest rate, r_1, to maximize the indirect utility function of households in country C. At the optimum it must be the case that

$$\frac{\partial V(r_1)}{\partial r_1} = 0.$$

or

$$\begin{aligned} \frac{2}{(1+2r_1)} - \frac{1}{(1+r_1)} - \frac{1}{(2-r_1)} &= 0 \\ r_1^2 + 2r_1 - \frac{1}{2} &= 0 \end{aligned}$$

Solving this quadratic expression for r_1 we obtain two values

$$-1 \pm \sqrt{\frac{3}{2}}$$

We can discard the root that implies a value for the interest rate below -1, because an interest rate cannot be below -100 percent. So the only economically sensible solution is

$$r_1 = -1 + \sqrt{\frac{3}{2}} = 0.22$$

Let r_1^{cc} denote the world interest rate under optimal capital controls, and r^{free} the world interest rate under capital mobility. Recall that r^{free} equals 1/3. Then we have that

$$r^{cc} < r^{free}. \tag{6.17}$$

We have shown that under optimal capital controls the world interest rate is lower than under free capital mobility. It follows directly from the fact that the current account schedule of country US is increasing in the world interest rate, see equation (6.5), that when country C manipulates its current account, the current account balance of country US deteriorates and hence the current account balance of country C improves, i.e., is larger than it would be in the absence of manipulation. From here we can deduce immediately that period 1 consumption in country C is lower than it would be under free capital mobility and that period-1 consumption in country US is higher than it would be under free capital mobility.

How can the government of country C induce individual households to reduce consumption in period 1? In order for households to consume less in period 1 it must be the case that the domestic interest rate in country C is higher than it would have been under free capital mobility. Let r_1^{c*} denote the domestic interest rate in country C when capital controls are in effect. Use the fact that households in country C still can borrow and lend at the rate r_1^{c*}. (But capital cannot flow freely across borders.) Then we know from household optimization that the marginal rate of substitution must be equal to the domestic interest rate. Recall that the optimal consumption bundle has the property that the marginal rate of substitution is equal to the interest rate:

$$1 + r_1^{c*} = MRS = \frac{U_1(C_1^C, C_2^C)}{U_2(C_1^C, C_2^C)} = \frac{C_2^C}{C_1^C}$$

If we knew the level of consumption under optimal capital controls we would know the value of the domestic interest rate, r_1^{c*}. We can find the level of consumption from equations (6.15) and (6.16) which give consumption in terms of the world interest rate

$$\begin{aligned} 1 + r_1^{c*} &= \frac{C_2^C}{C_1^C} \\ &= \frac{\frac{1}{2}Q[2 - r_1^{cc}]}{\frac{1}{2}Q\frac{1+2r_1^{cc}}{1+r_1^{cc}}} \\ &= \frac{3}{2} \\ &= 1.5 \end{aligned}$$

Recalling that the world interest rate under capital controls, r^{cc}, is 0.22, and that the world interest rate under free capital mobility, r^{free}, is 0.33, we have that

$$r_1^{cc} < r_1^{free} < r_1^{c*}$$

This result says that under optimal capital controls the world interest rate falls from 33 percent to 22 percent, but for the real allocation which is associated with this amount of (lower) international borrowing from country C, it must be the case that the domestic interest rate in country C, due to capital controls, goes up to 50 percent. The wedge between domestic and international interest rates is 50-22, or 28 percent.

Can we show that lifetime utility in country C is higher under current account manipulation (and what about lifetime utility in country US. Is it lower?) To find the level of utility under optimal capital controls we need to construct the product $C_1^C C_2^C$. Above we obtained C_1^C and C_2^C as a function of the interest rate. Using these two expressions, we have

$$U(C_1^C, C_2^C) = \ln\left(C_1^C C_2^C\right) = \ln\left(\frac{1}{2}Q\frac{1+2r_1^{cc}}{1+r_1^{cc}}\frac{1}{2}Q(2-r_1^{cc})\right)$$

Now use (6.17) to obtain:

$$U(C_1^C, C_2^C) = \ln\left(\frac{Q^2}{4}(7-2\sqrt{6})\right) = \ln\left(\frac{25.2122}{48}Q^2\right)$$

Recall that lifetime utility in country C under free capital mobility was equal to $\ln\left(\frac{25}{48}Q^2\right)$. As expected under optimal capital control, utility is higher than under free capital mobility.

By what fraction, λ do we need to increase period 1 and period 2 consumption under free mobility so as to make residents of country C as well off under free capital mobility as under optimal current account manipulation. The answer is $\lambda = \sqrt{\frac{12(7-2\sqrt{6})}{25}} - 1 = 0.0042$. So we need to increase consumption in both periods by (only) four tenth of one percent.

What about utility in country US. We expect that the optimal current account manipulation of country C is welfare decreasing for country US. The current account manipulation by country C forces country US to consume more in period 1 and less in period 2. Formally, we have

$$C_1^{US} = \frac{3}{2}Q - \frac{1}{2}Q\frac{1+2r_1^{cc}}{1+r_1^{cc}} = \frac{Q}{2}\frac{2+r_1^{cc}}{1+r_1^{cc}} = Q\left(\frac{1}{2} + \frac{1}{\sqrt{6}}\right)$$

$$C_2^{US} = 2Q - \frac{1}{2}Q(2-r_1^{cc}) = \frac{1}{2}Q(2+r_1^{cc}) = Q\left(\frac{1}{2} + \frac{\sqrt{6}}{4}\right)$$

$$U(C_1^{US}, C_2^{US}) = \ln\left(\frac{Q}{2}\frac{2+r_1^{cc}}{1+r_1^{cc}}\frac{Q}{2}(2+r_1^{cc})\right) = \ln\left(Q^2\frac{24+10\sqrt{6}}{48})\right) = \ln\left(1.0103Q^2\right)$$

Under free capital mobility, utility in country US was higher, $\ln\left(1.0208Q^2\right)$, see equation (6.13). But notice, that it continues to be the case that even when country C manipulates the current account, country US benefits from trading with country C. For under financial autarky lifetime utility would be only $\ln Q^2$.

To induce the increase in the domestic interest rate in country C, r_1^{c*}, the government of this country could levy a tax on international borrowing equal to the difference between r_1^{c*} and the world interest rate r_1. Such a tax is called a capital control, or Tobin, tax. For this tax not to have an income effect, the government must rebate in a non-distorting fashion (e.g., with a lump sum transfer) any revenues it collects with the capital control. In this way the tax only affects the allocation of consumption over time.

6.7 Exercises

1. **A Two-Country Economy**

   Consider a two-period, two-country, endowment economy. Let one of the countries be the United States and the other Europe. Households in the United States have preferences described by the utility function

   $$\ln C_1^U + \ln C_2^U,$$

   where C_1^U and C_2^U denote consumption of U.S. households in periods 1 and 2, respectively. Europeans have identical preferences, given by

   $$\ln C_1^E + \ln C_2^E,$$

   where C_1^E and C_2^E denote consumption of European households in periods 1 and 2, respectively. Let Q_1^U and Q_2^U denote the U.S. endowments of goods in periods 1 and 2, respectively. Similarly, let Q_1^E and Q_2^E denote the European endowments of goods in periods 1 and 2, respectively. Assume further that the endowments are nonstorable, that the U.S. and Europe are of equal size, and that there is free capital mobility between the two economies. The United States starts period 1 with a zero net foreign asset position carried over from period 0.

   (a) **Symmetric Equilibrium** Suppose that $Q_1^U = Q_2^U = Q_1^E = Q_2^E = 10$. Calculate the equilibrium world interest rate, and the current accounts in the United States and Europe in period 1.

(b) **US-Originated Contraction # 1** Suppose that a contraction originates in the United States. Specifically, assume that Q_1^U drops from 10 to 8. All other endowments (Q_2^U, Q_1^E, and Q_2^E) remain unchanged at 10. This contraction in output has two characteristics: First, it originates in the United States (the European endowments are unchanged.) Second, it is temporary (the U.S. endowment is expected to return to its normal value of 10 after one period). Calculate the equilibrium interest rate and the two current accounts in period 1. Provide intuition.

(c) **US-Originated Contraction # 2** Suppose now a second type of contraction in which the U.S. endowment falls from 10 to 8 in the first period but is expected to continue falling to 6 in the second period ($Q_1^U = 8$ and $Q_2^U = 6$). The endowments in Europe remain unchanged at 10 each period ($Q_1^E = Q_2^E = 10$). Like the one described in the previous item, this contraction originates in the United States. However, it differs from the one described in the previous paragraph in the fact that it displays a more protracted string of negative output growth rates. Calculate again the equilibrium interest rate and the two current accounts in period 1. Point out differences in the effects of the two types of contraction and provide intuition.

(d) At the beginning of the great contraction of 2008, interest rates fell sharply around the world. What does the model above say about people's expectations around 2008 about the future path of real activity.

Chapter 7

Twin Deficits: Fiscal Deficits and Current Account Imbalances

The model economies we have studied thus far feature two types of agents, households and firms. These models leave out the government. This is an important omission. For the government is a large economic agent controlling through taxes, transfers, public consumption, and public investment at least a third of economic activity in most developed and emerging market oriented economies. In this chapter, we investigate the role of the government in the determination of the current account.

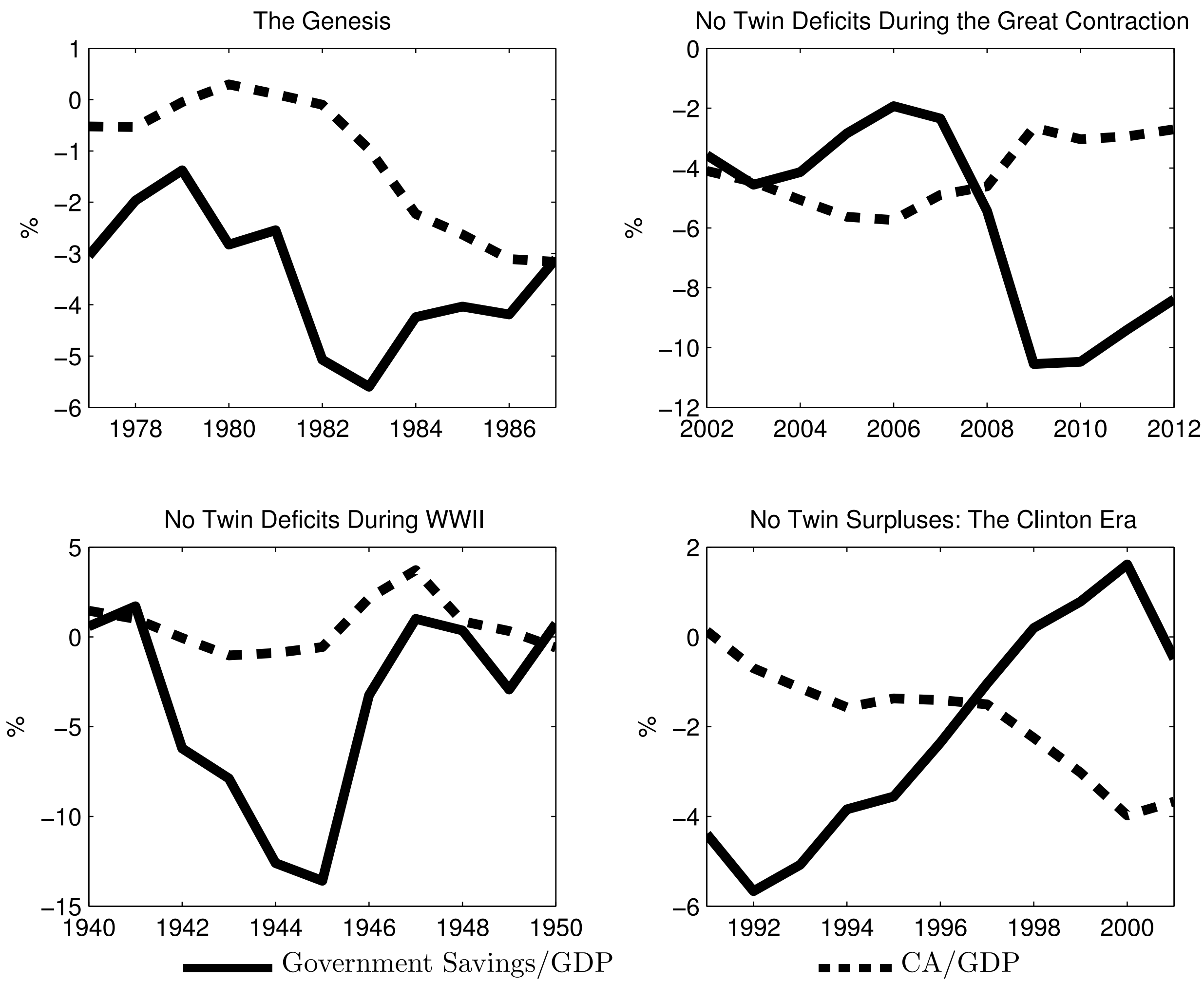
Our discussion will be centered around the so-called twin-deficits hypothesis, according to which fiscal deficits lead to current account deficits. In a nut shell, the idea behind the twin deficit hypothesis is as follows. Start with the definition of the current account as the difference between national savings and aggregate investment. In turn, national savings is the sum of private savings and government savings (or fiscal surpluses). Suppose now that expansionary government spending lowers government savings. If private savings and investment are unaffected by the expansionary fiscal policy, then the current account must deteriorate by the same amount as the decline in government savings.

7.1 Twin Deficits in the United States

In previous chapters, we have documented that the early 1980s were a turning point for the U.S. current account. Until 1982, the U.S. had run current account surpluses but thereafter a string of large current account deficits opened up. The emergence of large current account deficits coincided with large fiscal deficits that were the result of the Reagan administration's policy of tax cuts and increases in military spending. The joint deterioration of the current account and the fiscal balance that took place in the early 1980s is documented in the top left panel of figure 7.1

Are twin deficits a recurrent phenomenon? To answer this question, it is of interest to look at other episodes of large changes in government savings. The most recent episode of this type is the fiscal stimulus plan implemented by the Obama administration in the wake of the Great contraction of 2007. The Obama fiscal stimulus plan resulted in the largest fiscal deficits (as a fraction of GDP) in the postwar United States. The top right panel of figure 7.1 shows that between 2007 and 2009, the fiscal deficit of the

Figure 7.1: The Twin-Deficit Hypothesis in the United States



Data Source: `bea.gov`

United States increased by 8 percentage points of GDP. During the same period, however, contrary to the predictions of the twin-deficit hypothesis, the current account improved by about 2.5 percent of GDP.

In addition to the Reagan and Obama fiscal expansions, two other episodes stand out. One is the enormous albeit short-lived fiscal deficit during the second world war of about 12 percent of GDP, caused primarily by military spending (see the bottom left panel of figure 7.1). During this period, the current account did deteriorate from about 1 percent to -1 percent of GDP. This movement in the external account is in the direction of the twin-deficit hypothesis. However, the observed decline in the current account balance was so small relative to the deterioration in government savings, that the episode can hardly be considered one of twin deficits. Another noticeable change in the fiscal balance took place in the 1990s during the Clinton administration. Between 1990 and 2000, government savings increased by about 7 percentage points of GDP. At the same time, contrary to the twin-deficit hypothesis, the current account deteriorated by about 4 percent of GDP. In summary, over the past century large changes in government savings have not always been accompanied by equal adjustments in the current account.

7.2 Testable Implications of the Twin Deficit Hypothesis

The fact that there seems to be no systematic relationship between large changes in government savings and changes in the current account does not necessarily invalidate the twin-deficit hypothesis. In reality, economies are hit simultaneously by a multitude of shocks of different nature. As a result, it is difficult to infer from raw data, like that presented in figure 7.1, the effect of an increase in the fiscal deficit on the current account.

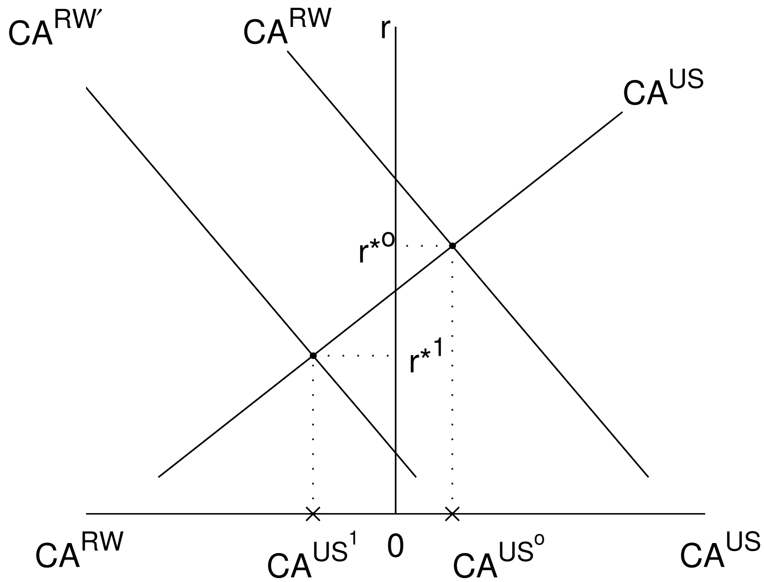
What then led some economists to conclude that the Reagan fiscal deficits were the cause of the current account deficits? To answer this question, we need to look at the implications that the twin-deficit hypothesis has for the behavior of variables other than the current account and the fiscal deficit and then compare those predictions to actual data.

In the early 1980s not all economic observers attributed the emergence of current accounts deficits to the fiscal stance. There were two prevailing theoretical views on the source of current account deficits.

One view was that in those years the rest of the world wanted to send their savings to the U.S., so the U.S. had to run a current account deficit. This view is illustrated in figure 7.2. The increase in the rest of the world's demand for U.S. assets is reflected in a shift to the left of the current account schedule of the rest of the world. As a result, in the new equilibrium position, the current account in the U.S. deteriorates from CA^{US^0} to CA^{US^1} and the world interest rate falls from r^{*0} to r^{*1}.

What could have triggered such an increase in the desire of the rest of the world to redirect savings to the U.S.? A number of explanations have been offered. First, in the early 1980s, the U.S. was perceived as a "safe heaven," that is, as a safer place to invest. This perception triggered an increase in the supply of foreign lending. For example, it has been argued that international investors were increasingly willing to hold U.S. assets due to instability in Latin America; in the jargon of that time, the U.S. was the recipient of the

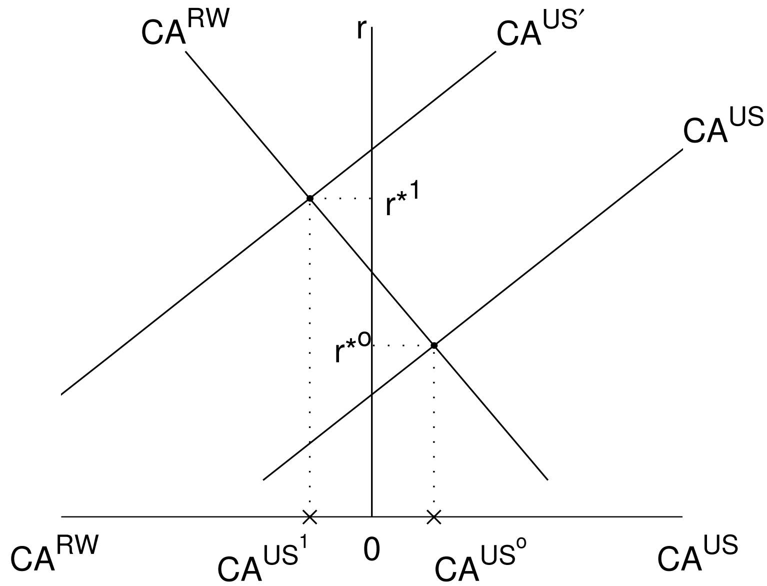
Figure 7.2: The U.S. current account in the 1980s: view 1



"capital flight" from Latin America. Second, as a consequence of the debt crisis of the early 1980s, international credit dried up, forcing developing countries, particularly in Latin America, to reduce current account deficits. Third, financial deregulation in several countries made it easier for foreign investors to hold U.S. assets. An example is Japan in the late 1980s.

A second view of what caused the U.S. current account deficit is that in the 1980s the U.S. wanted to save less and spend more at any level of the interest rate. As a result, the American economy had to draw savings from the rest of the world. Thus, U.S. foreign borrowing went up and the current account deteriorated. Figure 7.3 illustrates this view. As a result of the increase in desired spending relative to income in the U.S., the CA schedule for the U.S. shifts to the left, causing a deterioration in the U.S. current account from CA^{US^0} to CA^{US^1} and an increase in the world interest rate

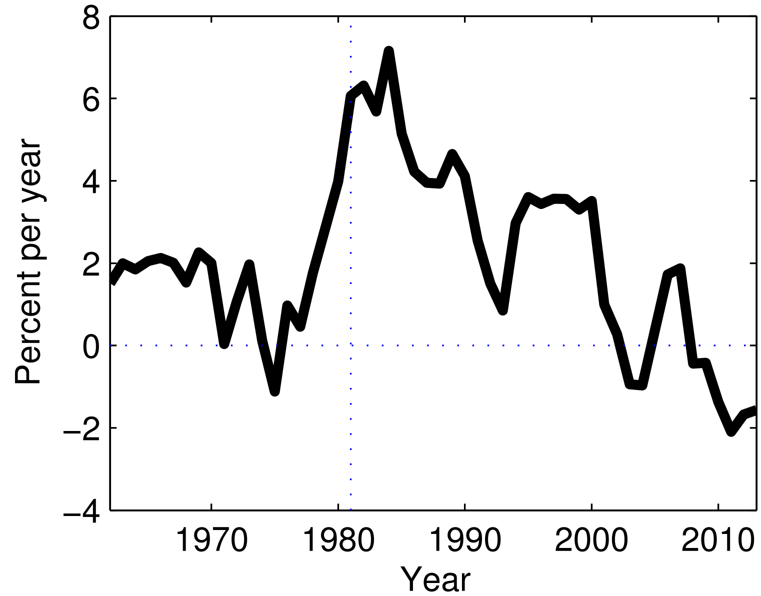
Figure 7.3: The U.S. current account in the 1980s: view 2



from r^{*0} to r^{*1}. Under view 2, the deterioration of the U.S. current account is the consequence of a decline in U.S. national savings or an increase in U.S. investment or a combination of the two.

How could we tell views 1 and 2 apart? One strategy is to look for an economic variable about which the two views have different predictions. Once we have identified such a variable, we could look at actual data to see which view its behavior supports. Comparing figures 7.2 and 7.3, it is clear that a good candidate for testing the two views is the real interest rate. The two views have different implications for the behavior of the interest rate in the U.S. Under view 1, the interest rate falls as the foreign supply of savings increases, whereas under view 2 the interest rate rises as the U.S. demand for funds goes up. What does the data show? In the early 1980s,

Figure 7.4: Real interest rates in the United States 1962-2013



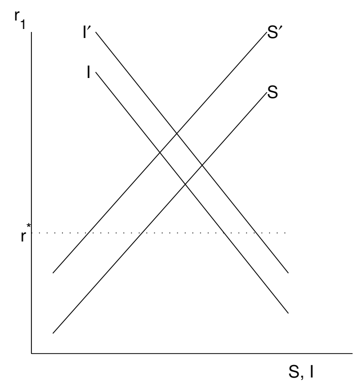
Note: The real interest rate is measured as the difference between the 1-year constant maturity Treasury rate and one-year expected inflation.

the U.S. experienced a large increase in real interest rates (see figure 7.4). This evidence seems to vindicate view 2. We will therefore explore this view further.

As already mentioned, view 2 requires that either the U.S. saving schedule shifts to the left, or that the U.S. investment schedule shifts to the right or both (see figure 7.5).

Before looking at actual data on U.S. savings and investment a comment about national savings is in order. National savings is the sum of private sector savings, which we will denote by S^p, and government savings, which

Figure 7.5: View 2 requires shifts in the U.S. savings or investment schedules



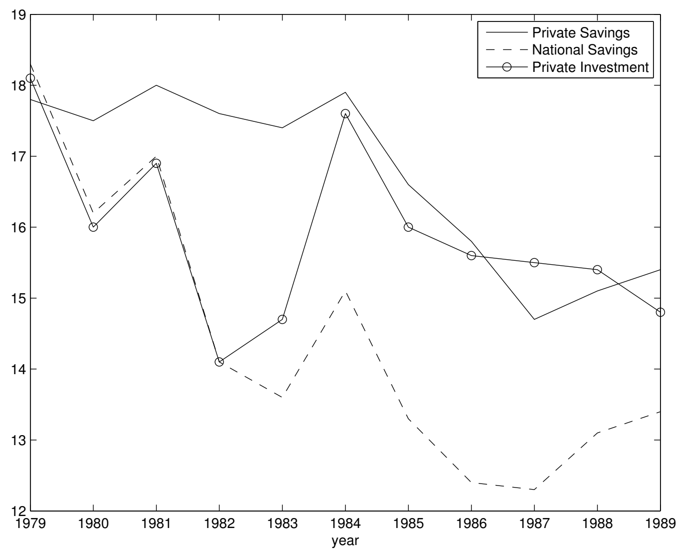
we will denote by S^g. Letting S denote national savings, we have

$$S = S^p + S^g.$$

Thus far we have analyzed a model economy without a public sector. In an economy without a government, national savings is simply equal to private savings, that is, $S = S^p$. However, in actual economies government savings accounts for a non-negligible fraction of national savings. To understand what happened to U.S. savings in the 1980s the distinction between private savings and government savings is important. With this comment in mind, let us now turn to the data.

Figure 7.6 displays with a solid line private savings, S^p, with a broken line national savings, S, and with a circled line investment, I. The difference between the solid and the broken lines represents government savings, S^g. The figure shows that national savings and private savings begin to diverge in 1980, with national savings falling consistently below private savings. This gap reflects the fiscal deficits created by the Reagan fiscal expansion.

Figure 7.6: U.S. Saving and Investment in Percent of GNP



Source: Norman S. Fieleke, "The USA in Debt," *New England Economic Review*, September-October 1990, pages 34-54, table 11.

Specifically, the increase in the fiscal deficit in the early 1980s arose due to, among other factors, a tax reform, which reduced tax revenues, and an increase in defense spending.

Advocates of the twin-deficit hypothesis emphasize the fact that the decline in the current account balance, given by $S-I$ (the difference between the broken line and the circled line in figure 7.6), is roughly equal to the decline in government savings (given by the difference between the solid and the broken lines). They therefore conclude that the increase in the fiscal deficit *caused* the decline in the current account. However, this causal direction, which implies that that the increase in the government deficit, that is, a decline in government savings, shifted the U.S. savings schedule to the left is not necessarily correct. The reason is that changes in fiscal policy that cause the fiscal deficit to increase may also induce offsetting increases in private savings, leaving total savings—and thus the current account—unchanged. In order to understand the relation between fiscal deficits and private savings, in the next section, we extend our theoretical model to incorporate the government.

7.3 The government sector in the open economy

Consider the two-period endowment economy studied in chapter 3, but assume the existence of a government that purchases goods G_1 and G_2 in periods 1 and 2, respectively, and levies lump-sum taxes T_1 and T_2. In addition, assume that the government starts with initial financial assets in the amount of B_0^g. The government faces the following budget constraints in periods 1 and 2:

$$G_1 + (B_1^g - B_0^g) = r_0 B_0^g + T_1$$

$$G_2 + (B_2^g - B_1^g) = r_1 B_1^g + T_2$$

where B_1^g and B_2^g denote the amount of government asset holdings at the end of periods 1 and 2, respectively. The left-hand side of the first constraint represents the government's outlays in period 1, which consist of government purchases of goods and purchases of financial assets. The right-hand side represents the government's sources of funds in period 1, namely, tax revenues and interest income on asset holdings. The budget constraint in period 2 has a similar interpretation.

Like households, the government is assumed to be subject to a no-Ponzi-game constraint that prevents it from having debt outstanding at the end of period 2. This means that B_2^g must be greater or equal to zero. At the same time, a benevolent government—that is, a government that cares about the welfare of its citizens—would not find it in its interest to end period 2 with positive asset holdings. This is because the government will not be around in period 3 to spend the accumulated assets in ways that would benefit its constituents. This means that the government will always choose B_2^g to be less than or equal to zero. The above two arguments imply that

$$B_2^g = 0.$$

Combining the above three expressions, we obtain the following intertemporal government budget constraint:

$$G_1 + \frac{G_2}{1+r_1} = (1+r_0)B_0^g + T_1 + \frac{T_2}{1+r_1} \tag{7.1}$$

This constraint says that the present discounted value of government consumption (the left-hand side) must be equal to the present discounted value of tax revenues and initial asset holdings including interest (the right-hand side). Note that there exist many (in fact a continuum of) tax policies T_1 and T_2 that finance a given path of government consumption, G_1 and G_2, i.e., that satisfy the intertemporal budget constraint of the government given by (7.1). However, all other things equal, given taxes in one period, the above intertemporal constraint uniquely pins down taxes in the other period. In particular, a tax cut in period 1 must be offset by a tax increase in period 2. Similarly, an expected tax cut in period 2 must be accompanied by a tax increase in period 1.

The household's budget constraints are similar to the ones we derived earlier in chapter 3, but must be modified to reflect the fact that now households must pay taxes in each of the two periods. Specifically, the household's budget constraints in periods 1 and 2 are given by

$$C_1 + T_1 + B_1^p - B_0^p = r_0 B_0^p + Q_1$$

$$C_2 + T_2 + B_2^p - B_1^p = r_1 B_1^p + Q_2$$

We also impose the no-Ponzi-game condition

$$B_2^p = 0.$$

Combining these three constraints yields the following intertemporal budget constraint:

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^p + Q_1 - T_1 + \frac{Q_2 - T_2}{1+r_1} \tag{7.2}$$

This expression says that the present discounted value of lifetime consumption, the left-hand side, must equal the sum of initial wealth, $(1+r_0)B_0^p$, and the present discounted value of endowment income net of taxes, $(Q_1 - T_1) + (Q_2 - T_2)/(1+r_1)$. Note that the only difference between the above intertemporal budget constraint and the one given in equation (3.4) is that now $Q_i - T_i$ takes the place of Q_i, for $i = 1, 2$.

As in the economy without a government, the assumption of a small open economy implies that in equilibrium the domestic interest rate must equal the world interest rate, r^*, that is,

$$r_1 = r^*. \tag{7.3}$$

The country's net foreign asset position at the beginning of period 1, which we denote by B_0^*, is given by the sum of private and public asset holdings, that is,

$$B_0^* = B_0^p + B_0^g.$$

We will assume for simplicity that the country's initial net foreign asset position is zero:

$$B_0^* = 0. \tag{7.4}$$

Combining (7.1), (7.2), (7.3), and (7.4) yields,

$$C_1 + G_1 + \frac{C_2 + G_2}{1 + r^*} = Q_1 + \frac{Q_2}{1 + r^*}.$$

This intertemporal resource constraint represents the consumption possibility frontier of the economy. It has a clear economic interpretation. The left-hand side is the present discounted value of domestic absorption, which consists of private and government consumption in each period.[1] The right-hand side of the consumption possibility frontier is the present discounted value of domestic output. Thus, the consumption possibility frontier states that the present discounted value of domestic absorption must equal the present discounted value of domestic output.

Solving for C_2, the consumption possibility frontier can be written as

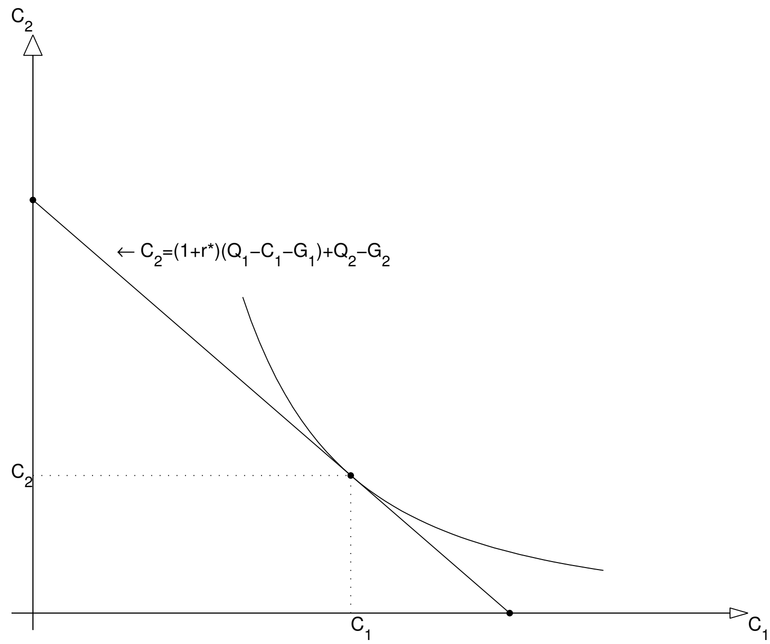
$$C_2 = (1 + r^*)(Q_1 - C_1 - G_1) + Q_2 - G_2. \tag{7.5}$$

Figure 7.7 depicts the relationship between C_1 and C_2 implied by the consumption possibility frontier. It is a downward sloping line with slope equal to $-(1 + r^*)$. Consumption in each period is determined by the tangency of the consumption possibility frontier with an indifference curve.

Note that neither T_1 nor T_2 appear in the consumption possibility fron-

[1] As noted in chapter 1, domestic absorption is the sum of consumption and investment. However, in the endowment economy under analysis investment is identically equal to zero.

Figure 7.7: Optimal consumption choice



tier. This means that, given G_1 and G_2, any combination of taxes T_1 and T_2 satisfying the government's budget constraint (7.1) will be associated with the same private consumption levels in periods 1 and 2.

7.4 Ricardian Equivalence

In order to understand the merits of the view that attributes the large current account deficits of the 1980s to fiscal deficits generated in part by the tax cuts implemented by the Reagan administration, we must determine how a reduction in taxes affects the current account in our model economy. Because the current account is the difference between national savings and investment, and because investment is by assumption nil in our endowment economy, it is sufficient to characterize the effect of tax cuts on national savings.[2] As mentioned earlier, national savings equals the sum of government savings and private savings.

Private savings in period 1, which we denote by S_1^p, is defined as the difference between *disposable income*, given by domestic output plus interest on net bond holdings by the private sector minus taxes, and private consumption:

$$S_1^p = Q_1 + r_0 B_0^p - T_1 - C_1.$$

Because, as we just showed, for a given time path of government purchases, private consumption is unaffected by changes in the timing of taxes and because $r_0 B_0^p$ is predetermined in period 1, it follows that changes in lump-sum taxes in period 1 induce changes in private savings of equal size and opposite sign:

$$\Delta S_1^p = -\Delta T_1. \tag{7.6}$$

The intuition behind this result is the following: Suppose, for example, that the government cuts lump-sum taxes in period 1, keeping government purchases unchanged in both periods. This policy obliges the government to increase public debt by ΔT_1 in period 1. In order to service and retire this additional debt, in period 2 the government must raise taxes by $(1+r_1)\Delta T_1$. Rational households anticipate this future increase in taxes and therefore choose to save the current tax cut (rather than spend it in consumption goods) so as to be able to pay the higher taxes in period 2 without having

[2]It is worth noting, however, that if the government levies only lump-sum taxes, as assumed in the present analysis, then the results of this section apply not only to an endowment economy but also to an economy with investment.

to sacrifice consumption in that period. Put differently, a change in the timing of lump-sum taxes does no alter the household's lifetime wealth.

Government savings, also known as the *secondary fiscal surplus*, is defined as the difference between revenues (taxes plus interest on asset holdings) and government purchases. Formally,

$$S_1^g = r_0 B_0^g + T_1 - G_1.$$

When the secondary fiscal surplus is negative we say that the government is running a *secondary fiscal deficit*. The secondary fiscal surplus has two components: interest income on government asset holdings ($r_0 B_0^g$) and the *primary fiscal surplus* ($T_1 - G_1$). The primary fiscal surplus measures the difference between tax revenues and government expenditures. When the primary fiscal surplus is negative, that is, when government expenditures exceed tax revenues, we say that the government is running a *primary deficit*.

Given an exogenous path for government purchases and given the initial condition $r_0 B_0^g$, any change in taxes in period 1 must be reflected one-for-one in a change in government saving, that is,

$$\Delta S_1^g = \Delta T_1. \tag{7.7}$$

As we mentioned before, national saving, which we denote by S, is given by the the sum of private and government saving, that is, $S_1 = S_1^p + S_1^g,$. Changes in national savings are thus equal to the sum of changes in private savings and changes in government savings,

$$\Delta S_1 = \Delta S_1^p + \Delta S_1^g.$$

Combining this expression with equations (7.6) and (7.7), we have that

$$\Delta S_1 = -\Delta T_1 + \Delta T_1 = 0.$$

This expression states that national savings is unaffected by the timing of lump-sum taxes. This is an important result in Macroeconomics. For this reason it has been given a special name: *Ricardian Equivalence*.[3]

Recalling that the current account is the difference between national saving and investment, it follows that the change in the current account in response to a change in taxes, holding constant government expenditure, is given by

$$\Delta CA_1 = \Delta S_1 - \Delta I_1.$$

Therefore, an increase in the fiscal deficit due to a decline in current lump-sum taxes (leaving current and expected future government spending unchanged) has *no* effect on the current account, that is,

$$\Delta CA_1 = 0.$$

[3] This important insight was first formalized by Robert Barro of Harvard University in "Are Government Bonds Net Wealth," *Journal of Political Economy,* 1974, volume 82, pages 1095-1117.

7.4.1 Then what was it?

Let us take stock of what we have learned from our model. If the model of Ricardian Equivalence represents an adequate description of how the economy works and if the main cause of the fiscal deficits of the 1980s was the Reagan tax cuts, then what we should have observed is a decline in public savings, an offsetting increase in private savings, and no change either in national savings or the current account. What does the data show? In the 1980s there was a significant cut in taxes. As predicted by theory, the tax cuts were accompanied by a significant decline in public savings (see the difference between the solid and broken lines in figure 7.6). However, contrary to the predictions of Ricardian Equivalence, private savings did not increase by the same amount as the decline in public savings and as a result both national savings and the current account to plummeted. We therefore conclude that either the fiscal deficits of the 1980s were caused by factors other than the tax cuts, such as increases in government spending, or Ricardian Equivalence does not hold, or both. We explore these possibilities further in the next section.

7.5 Government Spending and Current Account Deficits

What are other possible interpretations of the view according to which the large current account deficits of the 1980s were due to a decline in desired savings and/or an increase in desired U.S. spending? One possible interpretation is that the increase in the U.S. fiscal deficit of the 1980s was not solely a deferral of taxes, but instead government purchases were increased temporarily, particularly military spending. In our model, an increase in government purchases in period 1 of ΔG_1, with government purchases in period 2 unchanged, is equivalent to a temporary decline in output. In response to the increase in government spending, households will smooth consumption by reducing consumption spending in period 1 by less than the increase in government purchases ($\Delta C_1 + \Delta G_1 > 0$). Because neither output in period 1 nor investment in period 1 are affected by the increase in government purchases, the trade balance in period 1, which is given by $Q_1 - C_1 - G_1 - I_1$, deteriorates ($\Delta TB_1 = -\Delta C_1 - \Delta G_1 < 0$). The current account, given by $r_0 B_0^* + TB_1$, declines by the same amount as the trade balance ($\Delta CA_1 = \Delta TB_1$; recall that net investment income is predetermined in period 1). The key behind this result is that consumption falls by less than the increase in government purchases. The effect of the increase in government purchases on consumption is illustrated in figure 7.8. The initial consumption allocation is point A. The increase in G_1 produces a parallel shift in the economy's resource constraint to the left by ΔG_1. If consumption in both periods is normal, then both C_1 and C_2 decline. Therefore, the new optimal allocation, point B, is located southwest of point A. Clearly, the decline in C_1 is less in absolute value than ΔG_1.

Is this explanation empirically plausible? There exists evidence that government spending went up in the early 1980s due to an increase in national defense spending as a percentage of GNP. Table 7.1 indicates that military purchases increased by about 1.5% of GNP from 1978 to 1985. But accord-

Figure 7.8: Adjustment to a temporary increase in government purchases

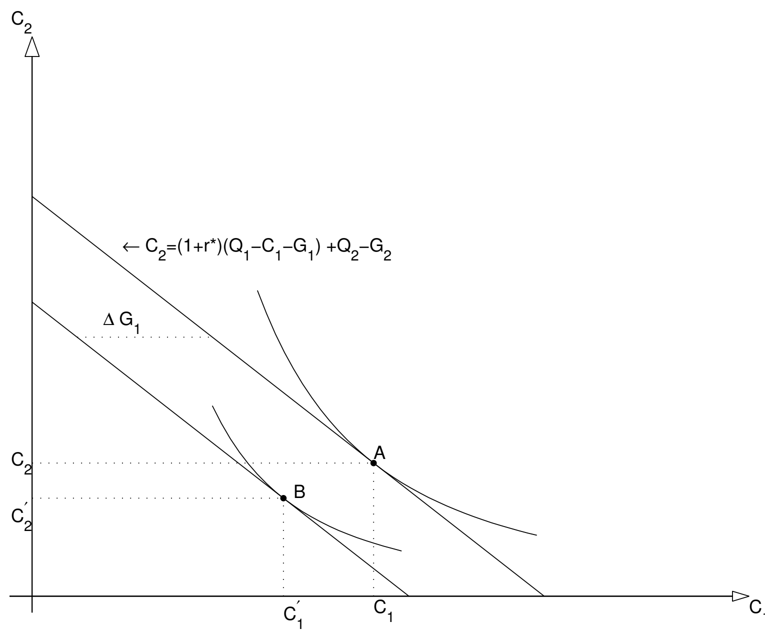


Table 7.1: U.S. military spending as a percentage of GNP: 1978-1987

Year	Military Spending (% of GNP)
1978-79	5.1-5.2
1980-81	5.4-5.5
1982-84	6.1-6.3
1985-87	6.7-6.9

ing to our model, this increase in government purchases (if temporary) must be associated with a decline in consumption. Thus, the decline in national savings triggered by the Reagan military build up is at most 1.5% of GNP, which is too small to explain all of the observed decline in national savings of 3% of GNP that occurred during that period (see figure 7.6).

7.6 Failure of Ricardian Equivalence

Thus far, we have considered two arguments in support of the view that the US external imbalances of the 1980s were the result of a decline in domestic savings (view 2). One was increases in government spending and the other was cuts in taxes. We concluded that if Ricardian Equivalence holds, then cuts in taxes could not explain the observed deterioration in the U.S. current account. A third argument in support of view 2 is that Ricardian Equivalence may not be right.

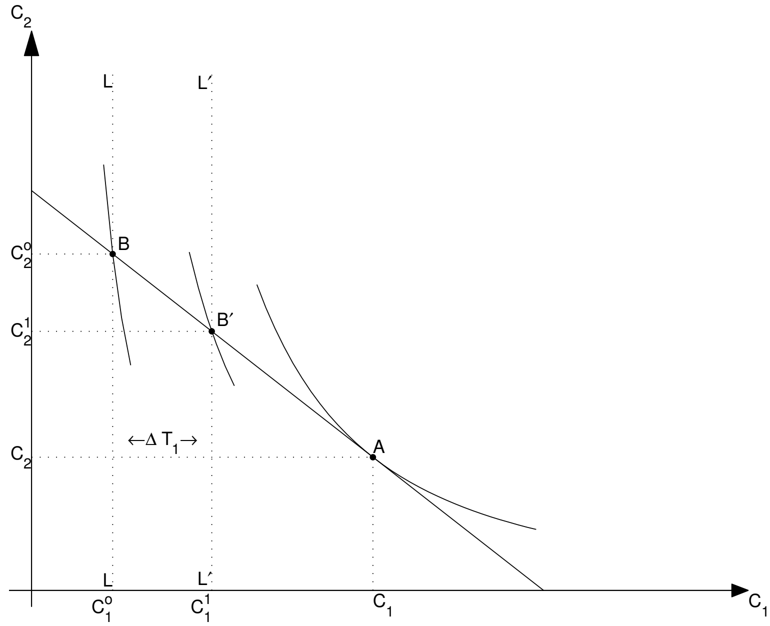
There are at least three reasons why Ricardian Equivalence may fail to hold. One is that households are borrowing constrained. A second reason is that the people that benefit from the tax cut are not the same that must pay for the future tax increase. And a third reason is that taxes are not lump-sum. In what follows of this section, we will explore each of these reasons in some detail.

7.6.1 Borrowing Constraints

To see why borrowing constraints may lead to a breakdown in Ricardian Equivalence, consider the case of a young worker who expects his future income to be significantly higher than his current income, perhaps due to on-the-job training or to the fact that he is simultaneously attending a good college. Based on this expectation, he might want to smooth consumption over time by borrowing against his higher future income. However, suppose that, perhaps because of imperfections in financial markets, such as asymmetric information between borrowers and lenders, he cannot procure a loan. In this case, the young worker is said to be borrowing constraint. Suppose now that the government decides to implement a cut in current (lump-sum) taxes, financed by an increase in future taxes. Will the young worker increase his savings by the same amount as the tax cut as prescribed by Ricardian Equivalence? Most likely not. He will probably view the tax cut as a welcome relief from his borrowing constraint and allocate it to consumption. So the decline in government savings due to the tax cut causes no changes in private savings. As a result, national savings and the current account will both deteriorate following the cut in lump-sum taxes.

Let's analyze this story a more formally. Suppose households have initial wealth equal to zero ($B_0^p = 0$) and that they are precluded from borrowing in financial markets, that is, they are constrained to choose $B_1^p \geq 0$. Assume further that neither firms nor the government are liquidity constrained, so that they can borrow at the world interest rate r^*. Figure 7.9 illustrates this case. Suppose that in the absence of borrowing constraints, the consumption allocation is given by point A, at which households in period 1 consume more than their after-tax income, that is, $C_1^0 > Q_1 - T_1$. This excess of consumption over disposable income is financed by borrowing in the financial market ($B_1^p < 0$). In this case the borrowing constraint is binding, and

Figure 7.9: Adjustment to a temporary tax cut when households are liquidity constrained



households are forced to choose the consumption allocation B, where $C_1 = Q_1 - T_1$. It is easy to see why, under these circumstances, a tax cut produces an increase in consumption and a deficit in the current account. The tax cut relaxes the household's borrowing constraint. The increase in consumption is given by the size of the tax cut ($\Delta C_1 = -\Delta T_1$), which in figure 7.9 is measured by the distance between the vertical lines L and L′. The new consumption allocation is given by point B', which lies on the economy's resource constraint and to the right of point B. Consumption in period 1 increases by the same amount as the tax cut. Because neither investment nor government purchases are affected by the tax cut, the trade balance and hence the current account deteriorate by the same amount as the increase in consumption. Thus, in the presence of borrowing constraints the increase in the fiscal deficit leads to a one-for-one increase in the current account deficit.

Can the presence of financial constraints *per se* explain the current account deficits of the 1980s as being a consequence of expansionary fiscal policy? The tax cut implemented during the Reagan administration amounted to about 3 percent of GDP. The observed deterioration in the current account during those years was also of about 3 percent of GDP. It is then clear that in order for the liquidity-constraint hypothesis alone to explain the behavior of the current account in the 1980s, it should be the case that 100% of the population must be borrowing constrained.

7.6.2 Intergenerational Effects

A second reason why Ricardian Equivalence could fail is that those who benefit from the tax cut are not the ones that pay for the tax increase later. To illustrate this idea, consider an endowment economy in which households live for only one period. Then, the budget constraint of the generation alive in period 1 is given by $C_1 + T_1 = Q_1$, and similarly, the budget constraint of the generation alive in period 2 is $C_2 + T_2 = Q_2$. Suppose that the government implements a tax cut in period 1 that is financed with a tax increase in period 2. Clearly, $\Delta C_1 = -\Delta T_1$ and $\Delta C_2 = -\Delta T_2$. Thus, the tax cut produces an increase in consumption in period 1 and a decrease in consumption in period 2. As a result, the trade balance and the current account in period 1 decline one-for-one with the decline in taxes. The intuition for this result is that in response to a decline in taxes in period 1, the generation alive in period 1 does not increase savings in anticipation of the tax increase in period 2 because it will not be around when the tax increase is implemented. What percentage of the population must be 1-period lived in order for this hypothesis to be able to explain the observed 3% of GNP decline in the U.S. current account balance, given the 3% decline in government savings? Obviously, everybody must be 1-period lived.

7.6.3 Distortionary Taxation

Finally, Ricardian equivalence may also breakdown if taxes are not lump sum. Lump-sum taxes are those that do not depend on agents' decicions. In the economy described in section 7.3, households are taxed T_1 in period 1 and T_2 in period 2 regardless of their consumption, income, or savings. Thus, in that economy lump-sum taxes do not distort any of the decisions of the households. In reality, however, taxes are rarely lump sum. Rather, they are typically specified as a fraction of consumption, income, firms' profits etc. Thus, changes in tax rates will tend to distort consumption, savings, and investment decisions. Suppose, for example, that the government levies a proportional tax on consumption, with a tax rate equal to τ_1 in period 1 and τ_2 in period 2. Then the after-tax cost of consumption is $(1+\tau_1)C_1$ in period 1 and $(1+\tau_2)C_2$ in period 2. In this case, the relative price of period-1 consumption in terms of period-2 consumption faced by households is not simply $1+r_1$, as in the economy with lump-sum taxes, but $(1+r_1)\frac{1+\tau_1}{1+\tau_2}$. Suppose now that the goverment implements a reduction in the tax rate in period 1. By virtue of the intertemporal budget constraint of the government, the public expects, all other things equal, an increase in the consumption tax rate in period 2. Thus, the relative price of current consumption in terms of future consumption falls. This change in the relative price of consumption induces households to substitute current for future consumption. Because firms are not being taxed, investment is not affected by the tax cut. As a result, the trade balance, given by $TB_1 = Q_1 - C_1 - G_1 - I_1$, and the current account, given by $CA_1 = TB_1 + r_0 B_0^*$, both deteriorate by the same amount.

We conclude that if the current account deficit of the 1980s is to be explained by the fiscal imbalances of the Reagan administration, then this explanation will have to rely on a combination of an increase in government expenditure and multiple factors leading to the failure of Ricardian equivalence.

7.7 Exercises

1. **An Small Open Economy With Distortionary Taxes**

   Consider a two-period economy populated by a large number of households with preferences described by the utility function

   $$\ln C_1 + \beta \ln C_2,$$

   where C_1 and C_2 denote consumption in periods 1 and 2, respectively, and $\beta = 1/1.1$ is a subjective discount factor. Households receive endowments Q_1 in period 1 and Q_2 in period 2, with $Q_1 = Q_2 = 10$ and can borrow or lend in international financial markets at the interest rate $r^* = 0.1$. The government imposes taxes $T_1 = T^L + \tau_1 C_1$ in period 1 and $T_2 = \tau_2 C_2$ in period 2 and consumes G_1 units of goods in period 1 and G_2 units in period 2. Finally, households and the government start period 1 with no assets or debts carried over from the past.

   (a) Derive the intertemporal budget constraint of the household, the intertemporal budget constraint of the government, and the intertemporal resource constraint of the economy as a whole.

   (b) Derive the optimality condition that results from choosing C_1 and C_2 to maximize the household's utility function subject to its intertemporal budget constraint.

   (c) Suppose $G_1 = G_2 = 2$ and $\tau_1 = \tau_2 = 0.2$. Find the equilibrium levels of consumption and the trade balance in periods 1 and 2, and the equilibrium level of lump-sum taxes T^L. Report the primary and secondary fiscal deficits in period 1.

(d) Continue to assume that $G_1 = G_2 = 2$. Suppose that the government implements a tax cut in period 1 consisting in lowering the consumption tax rate from 20 to 10 percent. Suppose further that lump-sum taxes, T^L, are kept at the level found in the previous item. Find consumption, the trade balance, the primary fiscal deficit in period 1, and the consumption tax rate in period 2.

(e) Now answer the previous question assuming that the cut in consumption taxes in period 1 from 20 to 10 percent is financed with an appropriate change in lump-sum taxes in the same period, while the consumption tax rate in period 2 is kept constant at its initial level of 20 percent. Compare your answer with the one for the previous item and provide intuition.

(f) Suppose that $G_1 = 2$, $G_2 = 1$, and $T^L = 0$. Clearly, there are many possible equilibrium tax schemes (τ_1, τ_2). Find the pair (τ^1, τ^2) that maximizes the household's lifetime utility. Show your derivation. Refer to your solution as the Ramsey optimal tax policy.

Chapter 8

International Capital Market Integration

Since the 1970s, a number of events around the world have made the assumption of free capital mobility increasingly realistic. Among the developments that have contributed to increased capital mobility are:

- The breakdown of the Bretton-Woods System of fixed exchange rates in 1972 allowed, as a byproduct, the removal of capital controls in some European countries, particularly in Germany in the mid 1970s.

- The high inflation rates observed in the 1970s together with the Federal Reserve's regulation Q which placed a ceiling on the interest rate that US banks could pay on time deposits, led to fast growth of eurocurrency markets. A eurocurrency deposit is a foreign currency deposit. For example, a Eurodollar deposit is a dollar deposit outside the United States (e.g., a dollar deposit in London). A yen deposit at a bank in Singapore is called a Euro yen deposit and the interest rate on such deposit is called the Euro yen rate (i.e., the interest rate on yen deposits outside Japan). The biggest market place for Eurocurrency deposits is London.

- Technological advances in information processing made it easier to watch several markets at once and to arbitrage instantly between markets.

- In the past few decades there has been a general trend for deregulation of markets of all kinds. For example, financial markets were deregulated in 1979 in Great Britain under the Thatcher administration and in the 1980s in the U.S. under the Reagan administration.

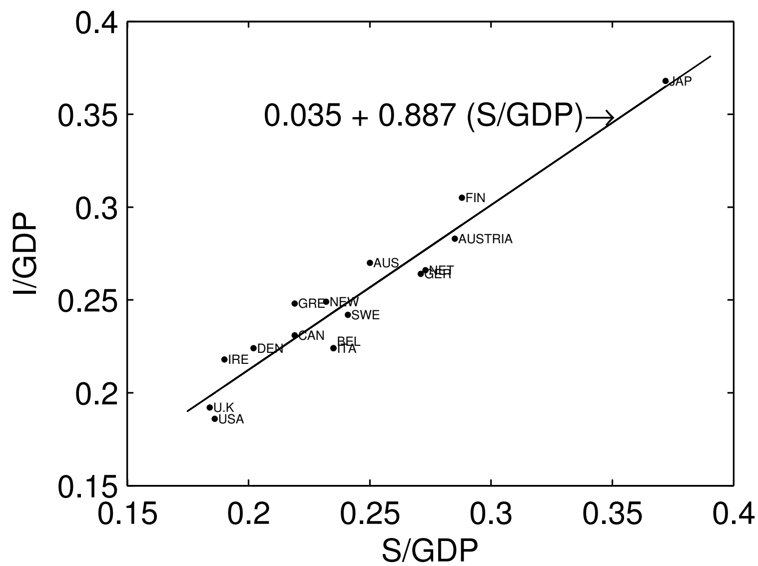
- In the 1980s and 1990s Europe underwent a process of economic and monetary unification. Specifically, capital controls were abolished in 1986, the single market became reality in 1992, and in 1999 Europe achieved a monetary union with the emergence of the Euro.

8.1 Measuring the degree of capital mobility: (I) Saving-Investment correlations

In 1980 Feldstein and Horioka wrote a provoking paper in which they showed that changes in countries' rates of national savings had a very large effect on their rates of investment.[1] Feldstein and Horioka examined data on average

[1] M. Feldstein and C. Horioka, "Domestic Saving and International Capital Flows," *Economic Journal* 90, June 1980, 314-29.

Figure 8.1: Saving and Investment Rates for 16 Industrialized Countries, 1960-1974 Averages



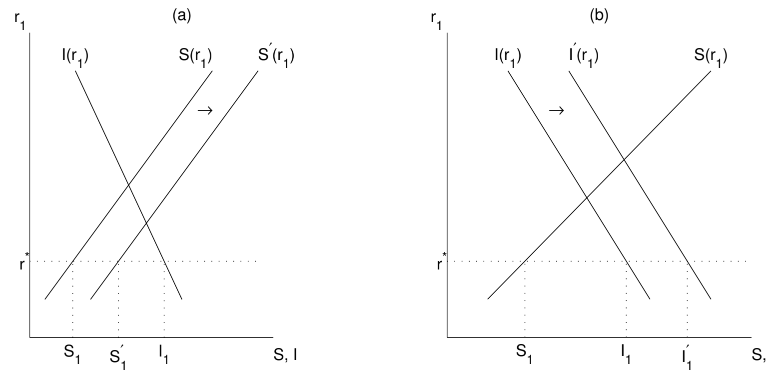
Source: M. Feldstein and C. Horioka, "Domestic Saving and International Capital Flows," *Economic Journal* 90, June 1980, 314-29.

investment-to-GDP and saving-to-GDP ratios from 16 industrial countries over the period 1960-74. The data used in their study is plotted in figure 8.1.

Feldstein and Horioka argued that if capital was highly mobile across countries, then the correlation between savings and investment should be close to zero, and therefore interpreted their findings as evidence of low capital mobility. The reason why Feldstein and Horioka arrived at this conclusion can be seen by considering the identity,

$$CA = S - I,$$

Figure 8.2: Response of S and I to independent shifts in (a) the savings schedule and (b) the investment schedule



where CA denotes the current account balance, S denotes national savings, and I denotes investment. In a closed economy—i.e., in an economy without capital mobility—the current account is always zero, so that $S = I$ and changes in national savings are perfectly correlated with changes in investment. On the other hand, in a small open economy with perfect capital mobility, the interest rate is exogenously given by the world interest rate, so that, if the savings and investment schedules are affected by independent factors, then the correlation between savings and investment should be zero. For instance, events that change only the savings schedule will result in changes in the equilibrium level of savings but will not affect the equilibrium level of investment (figure 8.2a). Similarly, events that affect only the investment schedule will result in changes in the equilibrium level of investment but will not affect the equilibrium level of national savings (figure 8.2b).

Feldstein and Horioka fit the following line through the cloud of points shown in figure 8.1:[2]

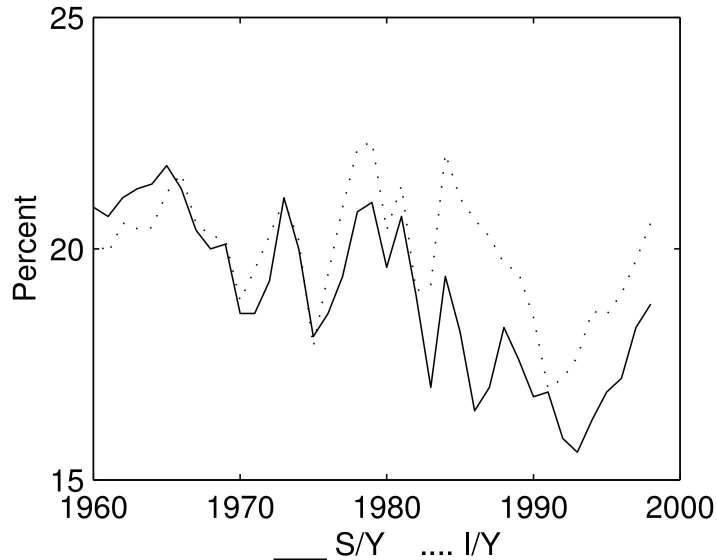
$$\left(\frac{I}{Q}\right)_i = 0.035 + 0.887\left(\frac{S}{Q}\right)_i + \nu_i; \qquad R^2 = 0.91$$

where $(I/Q)_i$ and $(S/Q)_i$ denote, respectively, the average investment-to-GDP and savings-to-GDP ratios in country i over the period 1960-74. Figure 8.1 shows the fitted relationship as a solid line. Feldstein and Horioka used data on 16 OECD countries, so that their regression was based on 16 observations. The high value of the coefficient on S/Q of 0.887 means that there is almost a one-to-one positive association between savings and investment rates. The reported R^2 statistic of 0.91 means that the estimated equation fits the data quite well, as 91 percent of the variation in I/Q is explained by variations in S/Q.

The Feldstein-Horioka regression uses cross-country data. A positive relationship between savings and investment rates is also observed within countries over time (i.e., in time series data). Specifically, for OECD countries, the average correlation between savings and investment rates over the period 1974-90 is 0.495. The savings-investment correlation has been weakening overtime. Figure 8.3 shows the U.S. savings and investment rates from 1955 to 1987. Until the late 1970s savings and investment were moving closely together whereas after 1980 they drifted apart. As we saw earlier (see figure 7.6), in the first half of the 1980s the U.S. economy experienced a large decline in national savings. A number of researchers have attributed

[2]The slope and intercept of this line are found by minimizing the sum of the squared distances between the line and each data point. This way of fitting a line through a cloud of points is called Ordinary Least Square estimation, or simply OLS estimation.

Figure 8.3: U.S. National Saving, Investment, and the Current Account as a Fraction of GNP, 1960-1998



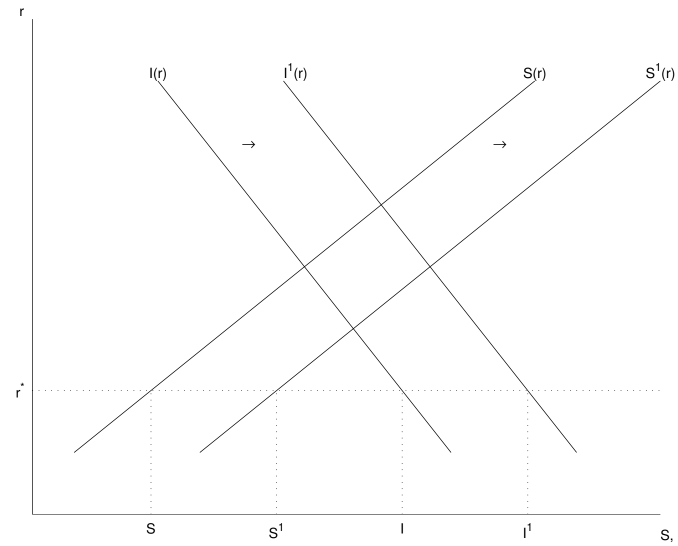
Source: Department of Commerce, Bureau of Economic Analysis, `www.bea.gov`.

the origin of these deficits to large fiscal deficits. Investment rates, on the other hand, remained about unchanged. As a result, the country experienced a string of unprecedented current account deficits. The fading association between savings and investment is reflected in lower values of the coefficient on S/Q in Feldstein-Horioka style regressions. Specifically, Frankel (1993)[3] estimates the relationship between savings and investment rates using time series data from the U.S. economy and finds that for the period 1955-1979 the coefficient on S/Y is 1.05 and statistically indistinguishable from unity. He then extends the sample to include data until 1987, and finds that the coefficient drops to 0.03 and becomes statistically indistinguishable from zero. In the interpretation of Feldstein and Horioka, these regression results show that in the 1980 the U.S. economy moved from a situation of very limited capital mobility to one of near perfect capital mobility.

But do the Feldstein-Horioka findings of high savings-investment correlations really imply imperfect capital mobility? Feldstein and Horioka's interpretation has been criticized on at least two grounds. First, even under perfect capital mobility, a positive association between savings and investment may arise because the same events might shift the savings and investment schedules. For example, suppose that, in a small open economy, the production functions in periods 1 and 2 are given by $Q_1 = A_1F(K_1)$ and $Q_2 = A_2F(K_2)$, respectively. Here Q_1 and Q_2 denote output in periods 1 and 2, K_1 and K_2 denote the stocks of physical capital (such as plant and equipment) in periods 1 and 2, $F(\cdot)$ is an increasing and concave production

[3]Jeffrey A. Frankel, "Quantifying International Capital Mobility in the 1980s," in D. Das, *International Finance*, Routledge, 1993.

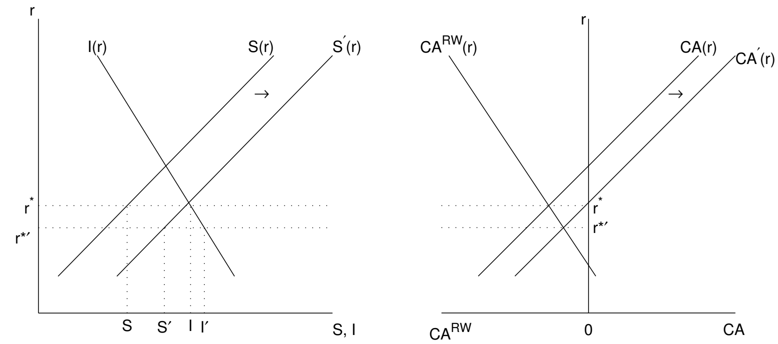
Figure 8.4: Response of S and I to a persistent productivity shock



function stating that the higher is the capital input the higher is output, and A_1 and A_2 are positive parameters reflecting factors such as the state of technology, the effects of weather on the productivity of capital, and so forth. Consider a persistent productivity shock. Specifically, assume that A_1 and A_2 increase and that A_1 increases by more than A_2. This situation is illustrated in figure 8.4, where the initial situation is one in which the savings schedule is given by $S(r)$ and the investment schedule by $I(r)$. At the world interest rate r^*, the equilibrium levels of savings and investment are given by S and I. In response to the expected increase in A_2, firms are induced to increase next period's capital stock, K_2, to take advantage of the expected rise in productivity. In order to increase K_2, firms must invest more in period 1. Thus, I_1 goes up for every level of the interest rate. This implies that in response to the increase in A_2, the investment schedule shifts to the right to $I^1(r)$. At the same time, the increase in A_2 produces a positive wealth effect which induces households to increase consumption and reduce savings in period 1. As a result, the increase in A_2 shift the savings schedule to the left. Now consider the effect of the increase in A_1. This should have no effect on desired investment because the capital stock in period 1 is predetermined. However, the increase in A_1 produces an increase in output in period 1 ($\Delta Q_1 > 0$). Consumption-smoothing households will want to save part of the increase in Q_1. Therefore, the effect of an increase in A_1 is a rightward shift in the savings schedule. Because we assumed that A_1 increases by more than A_2, on net the savings schedule is likely to shift to the right. In the figure, the new savings schedule is given by $S^1(r)$. Because the economy is small, the interest rate is unaffected by the changes in A_1 and A_2. Thus, both savings and investment increase to S^1 and I^1, respectively.

A second reason why savings and investment may be positively correlated in spite of perfect capital mobility is the presence of large country effects. Consider, for example, an event that affects only the savings schedule in a large open economy like the one represented in figure 8.5. In response to a shock that shifts the savings schedule to the right from $S(r)$ to $S'(r)$ the current account schedule also shifts to the right from $CA(r)$ to $CA'(r)$. As a result, the world interest rate falls from r^* to $r^{*\prime}$. The fall in the interest rate leads to an increase in investment from I to I'. Thus, in a large open economy, a shock that affects only the savings schedule results in positive comovement between savings and investment.

Figure 8.5: Large open economy: response of S and I to a shift in the savings schedule



8.2 Measuring capital mobility: (II) Interest rate differentials

A more direct measure of the degree of international capital mobility than the one used by Feldstein and Horioka is given by differences in interest rates across countries. In a world that enjoys perfect capital mobility, the rate of return on financial investments should be equalized across countries. Otherwise, arbitrage opportunities would arise inducing capital to flow out of the low-return countries and into the high-return countries. This movement of capital across national borders will tend to eliminate the difference in interest rates. If, on the other hand, one observes that interest rate differential across countries persist over time, it must be the case that in some countries restrictions on international capital flows are in place. It follows that a natural empirical test of the degree of capital market integration is to look at cross-country interest rate differentials. However, such a test is not as straightforward as it might seem. One difficulty in measuring interest rate differentials is that interest rates across countries are not directly comparable if they relate to investments in different currencies. Suppose, for example, that the interest rate on a 1-year deposit in the United States is 6 percent and on a 1-year deposit in Mexico is 30 percent. This interest rate differential will not necessarily induce capital flows to Mexico. The reason is that if the Mexican peso depreciates sharply within the investment period, an investor that deposited his money in Mexico might end up with fewer dollars at the end of the period than an investor that had invested in the United States. Thus, even in the absence of capital controls, differences in interest rates might exist due to expectations of changes in the exchange rate or as a compensation for exchange rate risk. It follows that a meaningful measure of interest rate differentials ought to take the exchange rate factor into account.

8.2.1 Covered interest rate parity

Suppose an investor has 1 US dollar and is trying to decide whether to invest it domestically or abroad, say in Germany. Let i denote the US interest rate and i^* the foreign (German) interest rate. If the investor deposits his money in the US, at the end of the period he receives $1 + i$ dollars. How many dollars will he have if instead he invested his 1 dollar in Germany? In order to invest in Germany, he must first use his dollar to buy euros. Let S denote the spot exchange rate, defined as the dollar price of 1 Euro. The investor gets $1/S$ euros for his dollar. At the end of the investment period, he will receive $(1 + i^*)/S$ euros. At this point he must convert the euros into dollars. Let S' denote the spot exchange rate prevailing at the end of the investment period. Then the $(1 + i^*)/S$ euros can be converted into $(1 + i^*)S'/S$ dollars. Therefore, in deciding where to invest, the investor compares the return of investing in the US, $1 + i$, to the dollar return of an equivalent investment in Germany, $(1 + i^*)S'/S$. If $1 + i$ is greater than $(1 + i^*)S'/S$, then it is more profitable to invest in the United States. In fact, in this case, the investor could make unbounded profits by borrowing in Germany and investing in the US. Similarly, if $1+i$ is less than $(1+i^*)S'/S$, the investor could make infinite profits by borrowing in the US and investing in Germany. This investment strategy suffers, however, from a fundamental problem. Namely, the fect that at the time the investment is made the exchange rate prevailing at the end of the investment period, S', is unknown. This means that the return associated with investing in the United States, $1 + i$, and the one associated with investing in Germany, $(1 + i^*)S'/S$, are not directly comparable because the former is known with certainty at the time the investment is made whereas the latter is uncertain at that time.

Forward exchange markets are designed precisely to allow investors to circumvent this problem. The investor can eliminate the exchange rate uncertainty by arranging at the beginning of the investment period, the purchase of the necessary amount of U.S. dollars to be delivered at the end of the investment period for a price determined at the beginning of the period. Such a foreign currency purchase is called a *forward contract*. Let F denote the forward rate, that is, the dollar price at the beginning of the investment period of 1 euro delivered and paid for at the end of the investment period. Then, the dollar return of a one-dollar investment in Germany using the forward exchange market is $(1 + i^*)F/S$. This return is known with certainty at the beginning of the investment period, making it comparable to the return on the domestic investment, $1+i$. The difference between the domestic return and the foreign return expressed in domestic currency by use of the forward exchange rate is known as the covered interest rate differential:

$$\text{Covered Interest Rate Differential } = (1+i) - (1+i^*)\frac{F}{S}.$$

This interest rate differential is called covered because the use of the forward exchange rate covers the investor against exchange rate risk.

The percentage difference between the forward exchange rate, F, and the spot exchange rate, S, is called the *forward discount*, which we will denote that fd. That is, $fd = (F-S)/F$. When the forward discount is not too big, it can be well approximated by $f-s$, where $f = \ln(F)$ and $s = \ln(S)$. For instance, suppose that $F = 1.01$ and $S = 1$, so that $fd = 0.01$ or 1%. In this case, we have that $f - s = 0.00995$, which is close to 0.01, or 1%. Also, when both the foreign interest rate and the forweard discount are small, the return on the foreign-currency investment, $(1+i^*)F/S$, can be reasonably approximated by $1 + I^* + f - s$. For instance, suppose that the foreign interest rate is 3% and that the foreign discount is 1%. Then, we have that $(1+i^*)F/S = 1.03 \times 1.01 = 1.0403$, and $1+i^*+f-s = 1+0.03+0.0095 = 1.0395$. Using these approximations, we can write the covered interest rate differential as:

$$\text{Covered Interest Rate Differential } = i - i^* - (f-s).$$

Or, using the notation fd for the forward discount,

$$\text{Covered Interest Rate Differential } = i - i^* - fd. \tag{8.1}$$

The covered interest rate differential is also known as the *country risk premium*. When the covered interest rate differential, or country risk premium, is zero or very close to zero, we say that *covered interest rate parity* holds. In the absence of barriers to capital mobility, a violation of covered interest rate parity implies the existence of arbitrage opportunities. That is, the possibility of making unbounded amounts of profits by borrowing in one country and investing in another without taking on any risk. Consider the following example. Suppose that the annual nominal interest rate in the U.S. is 7% ($i = 0.07$), that the annual nominal interest rate in Germany is 3% ($i^* = 0.03$), that the spot exchange rate is \$0.5 per euro ($S = 0.5$), and that the 1-year forward exchange rate is \$0.51 per euro ($F = 0.51$). In this case, the forward discount is 2%, or $fd = \ln(0.51/0.50) \approx 0.02$. Thus, the covered interest rate differential is $2\% = 7\% - 3\% - 2\%$. In the absence of barriers to international capital mobility, this violation of covered interest parity implies that it is possible to make profits by borrowing in Germany, investing in the U.S., and buying euros in the forward market to eliminate the exchange rate risk. To see how one can exploit this situation consider the following sequence of trades. (1) borrow 1 euro in Germany. (2) exchange your euro in the spot market for \$0.5. (3) Invest the \$0.5 in U.S. assets. (4) buy 1.03 euros in the forward market (you will need this amount of euros to repay your euro loan including interest). Note that buying ruros

Table 8.1: Covered interest rate differentials for selected countries September 1982-January 1988 (in percent)

	$i - i^* - fd$	
	Mean	Std. Dev.
Germany	0.35	0.03
Switzerland	0.42	0.03
Mexico	-16.7	1.83
France	-1.74	0.32

The covered interest rate differential is measured by the domestic 3-month interest rate minus the 3-month Euro-dollar interest rate minus the forward discount. Source: J. Frankel, *"Quantifying International Capital Mobility in the 1980s,"* in D. Das, *International Finance*, Routledge, 1993, table 2.6.

in the forward market involves no payment at this point. (5) After 1 year, your U.S. investment yields $1.07 \times \$0.5 = \$0.535$. (6) Execute your forward contract, that is, purchase 1.03 euros for $0.51\$/E \times E1.03 = \$0.5253$. The difference between what you receive in (5) and what you pay in (6) is $\$0.535 - \$0.5253 = \$0.0097 > 0$. Note that this operation involved no risk (because you used the forward marke), needed no initial capital, and yielded a pure profit of \$0.0097. It is clear from this example that the covered interest rate differential, or country premium, should be zero if there are no barriers to capital flows.

Table 8.1 shows the average covered interest rate differential for four countries over the period 1982-1988. Over that period Germany and Switzerland had small country risk premia: less than 50 basis points on average. Thus, Germany and Switzerland appeared to be relatively open to international capital flows in the early 1980s. By contrast, Mexico had an enormous negative country risk premium of over 16 percent. The period 1982-1988 corresponds to the post debt crisis period, when the financial sector in Mexico was nationalized and deposits were frozen. During that period, investors wanted to take their capital out of Mexico, but were impeded by financial regulations. In France barriers to the movement of capital were in place until 1986, which explains the large average deviations from covered interest rate parity vis-a-vis the two other industrialized countries shown in the table. The fact that the country risk premia of France and Mexico are negative indicates that capital controls were preventing capital from flowing out of these countries.

Table 8.2 presents an alternative approach to computing covered interest rate differentials. It uses interest rate differentials between domestic deposit rates and Eurocurrency deposit rates. For example, it compares the interest rate on a French franc deposit in France to the interest rate on a French franc deposit outside France, say in London. Since both deposits are in French francs the exchange rate plays no role in comparing the two interest rates. The table provides further evidence suggesting that the presence of capital controls leads to deviations from covered interest rate parity. It shows differences between domestic interbank and the corresponding Euro currency interest rate for France, Italy, Germany, and Japan from 1982 to 1993. In general, interest rate differentials are lower after 1987. This is most evident for France, where important capital market deregulation took place in 1986. In Italy, the high differential observed between 1990 and 1992 reflects market fears that capital controls might be imposed to avoid realignment of the lira, as an attempt to insulate the lira from speculative attacks, like the one that

Table 8.2: International capital mobility in the 1990s
Domestic Interbank minus Eurocurrency 3-month interest rates: (in percent)

Country	1/1/82-1/31/87	2/1/87-6/30/90	7/1/90-5/31/92	6/1/92-4/30/93
France	-2.27	-0.11	0.08	-0.01
Italy	-0.50	0.29	0.56	0.36
Germany	0.17	0.05	-0.05	0.07
Japan	-0.07	-0.60	0.09	0.17

Source: M. Obstfeld, *"International Capital Mobility in the 1990s,"* in Kenen, *Understanding Interdependence: The Macroeconomics of the Open Economy,* Princeton University Press, 1995, table 6.1.

took place in August/September 1992. These violent speculative attacks, which affected a number of European economies, particularly, France, Sweden, Italy, and England, led to exchange rate realignments and a temporary suspension of the European Exchange Rate Mechanism (ERM) in September 1992. Once the ERM was reestablished, the lira interest rate differential falls as fears of capital controls vanish. Japan had large onshore/offshore differentials between February 1987 and June 1990, which were the result of the Bank of Japan's heavy use of administrative guidelines to hold interbank rates below offshore rates.

The empirical evidence we have examined thus far shows that countries that have little barriers to capital mobility also tend to have small country premia on assets with short maturities, typically 3 months. However, this finding also holds for assets with longer maturities. For example, the covered interest rate differential on five-year U.S. government bonds versus Japanese bonds averaged only 0.017 percentage points in the period

10/3/1985 to 7/10/1986, and the differential on 7-year bonds averaged only 0.053 percentage points. Over the same period, the mean differentials on 5-year bonds for Germany were 0.284 percentage points and 0.187 percentage points for Switzerland.[4] The magnitude of the covered interest rate differentials at these longer maturities is in line with those reported in table 8.1 for much shorter maturities, supporting the argument that under free capital mobility covered interest rate differentials should vanish.

8.2.2 Real interest rate differentials and capital market integration

In the two-period model developed in previous chapters, perfect capital mobility amounts to the domestic real interest rate r_1 being equal to the world interest rate r^*. This suggests that another way of testing for capital mobility could be to look at real interest rate differentials across countries. Table 8.3 shows real interest rate differentials, $r - r^*$, in the 1980s for four countries. The average real interest rate differential over the sample period was significantly different from zero and quite volatile, with the highest mean and standard deviation for Mexico, at the time a closed developing country. But there seems to be a puzzle in the data shown in the table. For example, open developed economies such as Switzerland and Germany had large negative real interest rate differentials, while France had a much smaller real interest rate differential despite the fact that it had significant capital controls in place over most of the sample period. This suggests that real

[4]See, H. Popper, "International Capital Mobility: direct evidence from long-term currency swaps," IFDP # 386, Board of Governors of the Federal Reserve System, September 1990.

Table 8.3: Real interest rate differentials for selected countries
September 1982-January 1988

	$r - r^*$	
	Mean	Std. Dev.
Germany	-1.29	0.65
Switzerland	-2.72	0.81
Mexico	-20.28	9.43
France	-0.48	0.72

Note: The real interest rate differential $(r - r^*)$ is measured by the local minus the Eurodollar 3-month real expost interest rate (that is, interest differential less realized inflation differential). Source: Jeffrey A. Frankel, "Quantifying International Capital Mobility in the 1980s," in D. Das, *International Finance*, Routledge, 1993, table 2.5.

interest rate differentials might not be such a good measure of international capital mobility.

As will become clear soon, in reality, real interest rate differentials are not good indicators of the degree of capital mobility. They represent a good measure of international capital mobility *only if* the relative price of consumption baskets across countries does not change over time and if there is no nominal exchange rate uncertainty or if people don't care about that kind of risk. The first two conditions are met in our simple two-period model. In that model, there is only one good, which is assumed to be freely traded across countries. Thus, the relative price of consumption baskets across countries is constant and equal to one. In addition in that model there is no uncertainty, and in particular no exchange rate risk.

To show that in actual data capital mobility need not imply a zero real interest rate differential, we decompose the real interest rate differential into three components. We begin by noting that the real interest rate is given by the difference between the nominal interest rate and expected inflation, that is,

$$r = i - \pi^e \tag{8.2}$$

where r denotes the real interest rate, i denotes the nominal interest rate, and π^e denotes expected inflation. This relationship is often referred to as the Fisher equation. A similar relation must hold in the foreign country, that is,

$$r^* = i^* - \pi^{*e},$$

where starred variables refer to variables in the foreign country. Taking the difference of the domestic and foreign Fisher equations, we obtain,

$$r - r^* = (i - i^*) + (\pi^{*e} - \pi^e)$$

We will manipulate this expression to obtain a decomposition of the real interest rate differential, $r - r^*$, into three terms reflecting: (i) the degree of capital mobility; (ii) nominal exchange rate risk; and (iii) expected changes in relative prices across countries. For illustrative purposes, let the U.S. be the domestic country and Germany the foreign country. As above, let S be the spot nominal exchange rate defined as the price of 1 euro in terms of U.S. dollars and let S^e be the nominal exchange rate expected to prevail next period. Also, let F denote the forward rate. Let s, s^e, and f denote, respectively, the logs of S, S^e, and F. Add and subtract $s + s^e + f$ to the

Table 8.4: Decomposition of the real interest rate differential for selected countries: September 1982 to January 1988

Country	$r - r^*$	$i - i^* - fd$ (1)	$f - s^e$ (2)	$s^e - s + \pi^{*e} - \pi^e$ (3)
Germany	-1.29	0.35	4.11	-6.35
Switzerland	-2.72	0.42	3.98	-8.35
France	-0.48	-1.74	7.47	-6.26
Mexico	-20.28	-16.47	6.04	-3.32

Note: Columns (1), (2), and (3) do not add up to $r - r^*$ because in constructing (2) and (3) s^e, which is not directly observable, was proxied by the actual one-period-ahead spot exchange rate. Source: J. Frankel, *"Quantifying International Capital Mobility in the 1980s,"* in D. Das, *International Finance*, Routledge, 1993, tables 2.5, 2.6, 2.8, and 2.9.

right hand side of the above expression and rearrange terms to get

$$r - r^* = (i - i^* - fd) + (f - s^e) + (s^e - s + \pi^{*e} - \pi^e), \tag{8.3}$$

where we use the fact that $f - s$ equals the forward discount fd. The first term on the right-hand side of this expression is the covered interest rate differential. This term is zero if the country enjoys free capital mobility. However, the above expression shows that the real interest rate differential may not be equal to the covered interest rate differential if the sum of the second and third terms on the right-hand side is different from zero. To the extent that the sum of these two terms deviates significantly from zero, the real interest rate differential will be a poor indictor of the degree of capital market integration. This point is illustrated in table 8.4, which shows the decomposition of the real interest rate differential for Germany, Switzerland, France, and Mexico.

We next discuss in more detail the factors that introduce a wedge between real and covered interest rate differentials. We begin by analyzing the second term on right-hand side of (8.3), $f - s^e$, which we will call *exchange risk premium*. Then we will study the meaning of the third term, $s^e - s + \pi^{*e} - \pi^e$, which is known as the *expected real depreciation*.

8.2.3 Exchange Risk Premium $(f - s^e)$

The exchange risk premium measures the percentage difference between the forward and the expected future spot exchange rates. It depends on the degree of uncertainty about future exchange rates as well as on people's attitudes towards risk. If there is no uncertainty about future exchange rates, then $S^e = F$ and the exchange risk premium is therefore zero. If investors are risk neutral, then all people care about is expected returns. In particular, if S^e is, say, higher than F, then people would find it advantageous to buy euros in the forward market, which yields an expected profit of $S^e - F > 0$. Thus, agents would demand unbounded amounts of forward euros, driving F up until it is equal to S^e. Consequently, under risk neutrality $F = S^e$, or the exchange risk premium is zero. But typically the exchange risk premium is not zero reflecting the fact that neither of the two aforementioned assumptions hold. For example, column (2) of table 8.4 shows an estimate of the average exchange rate risk premium for Germany, Switzerland, France and Mexico over the period September 1982 to January 1988 using monthly data. For all countries the exchange risk premium is positive and high, ranging from 4 percentage points for Switzerland to 7.5

percentage points for France.

8.2.4 Expected Real Depreciation, $s^e - s + \pi^{*e} - \pi^e$

The third term on the right-hand side of (8.3) is related to expected changes in the relative price of consumption baskets in the domestic (US) and the foreign (German) country. The relative price of a German consumption basket in terms of a US consumption basket is known as the real exchange rate. We will denote the real exchange rate by e. Formally, e is given by

$$e = \frac{S \cdot P^*}{P}, \tag{8.4}$$

where P^* is the euro price of a German consumption basket and P is the dollar price of a US consumption basket. An increase in e means that Germany becomes more expensive relative to the U.S.. In this case, we say that the U.S. dollar experiences a *real depreciation* because one needs more U.S. consumption baskets to purchase one German basket. Similarly, a decline in e is referred to as a *real appreciation* of the U.S. dollar. Letting p and p^* denote the logs of P and P^*, we have

$$\ln e = s + p^* - p$$

The expectation of the log of the real exchange rate next period is similarly given by

$$\ln e^e = s^e + p^{*e} - p^e,$$

where the superscript ^{e} denotes expected value next period. It follows from the above two expressions that

$$\ln e^e - \ln e = (s^e - s) + (p^{*e} - p^*) - (p^e - p).$$

The left-hand side of this expression is the expected percentage depreciation of the real exchange rate, which we will denote by $\%\Delta e^e$. The first term on the right-hand side is the expected depreciation of the spot (or nominal) exchange rate. The second and third terms represent, respectively, expected consumer price inflation in the foreign (German) and the domestic (US) economies, π^{*e} and π^e. Thus, we can express the expected percentage increase in e as

$$\%\Delta e^e = s^e - s + \pi^{*e} - \pi^e, \tag{8.5}$$

Using (**??**) and (8.5) we can write the real interest rate differential given in (8.3) as

$$r - r^* = (i - i^* - fd) + (f - s^e) + \%\Delta e^e \tag{8.6}$$

This expression says that the real interest rate differential can be decomposed into the country premium, the exchange risk premium, and the expected depreciation of the real exchange rate. We use the following terminology:

- If $i - i^* - fd > 0$, we say that the country premium is positive.
- If $f - s^e > 0$, we say that the exchange risk premium is positive.
- If $\%\Delta e^e > 0$, we say that the real exchange rate is expected to depre-

ciate.

As we mentioned earlier, the real exchange rate, $e \equiv SP^*/P$, is the relative price of a basket of consumption in the foreign country in terms of a basket of consumption in the domestic country. Suppose that the baskets of consumption in both countries contained only one good, say wheat, and that the good is freely traded between the two countries. Then the price of wheat in the U.S., P, must equal the dollar price of buying wheat in Germany, which is given by P^*, the price of wheat in German euros, times S, the nominal exchange rate; that is, $P = P^*S$. Thus, in this case the real exchange rate, e, is identically equal to 1 in every period. When $e = 1$, we say that ***purchasing power parity*** (PPP) holds. Clearly, if PPP holds, then the expected real depreciation, $\%\Delta e^e$, is equal to zero because the real exchange rate is always expected to be equal to 1. In the 2-period model we have been studying thus far, there is only one good, which is freely traded in world markets. Thus, in our model, PPP holds.

In reality, however, PPP does not hold. Column (3) of table 8.4 shows that the German mark experienced a real appreciation of 6.3% per year vis-a-vis the US dollar over the period September 1982 to January 1988. This means that a basket of consumption in Germany became more expensive than a basket of consumption in the United States over the period considered. A similar pattern emerges for the other countries included in the table. In fact, for Germany and Switzerland, which had free capital mobility in the period covered by the table, the expected real appreciation explains the observed negative real interest rate differential. This is because for these two economies, the country premium is negligible and the exchange risk premium was positive.

But why does PPP not hold? An important reason is that the assumption that all goods are freely traded across countries, which we used to construct the wheat example, is counterfactual. In the real world there is a large number of goods that are not traded internationally, such as haircuts, housing, ground transportation, and so forth. We refer to these goods as *nontradables*. Also, barriers to international trade, such as import tariffs and quotas, introduce a wedge between the domestic and foreign prices of goods and services. We will explore the factors affecting the determination of the real exchange rate in more detail in the next chapter.

We conclude this section by reiterating that the real interest rate differential, $r - r^*$, is in general not a true measure of international capital mobility. Capital mobility is better measured by deviations from covered interest rate parity ($i - i^* - fd$). In the 2-period model we studied in previous chapters, there is only one good in each period, which is freely traded across countries and there is no exchange rate uncertainty. Thus, in our model both the exchange risk premium and expected real depreciation are equal to zero. This means that our model represents a special case in which real interest rate parity implies free capital mobility.

8.3 Uncovered Interest Rate Parity

Earlier in this chapter we derived the covered interest rate parity condition and showed that deviations from covered interest rate parity can only occur when international capital markets are imperfectly integrated. That is, we interpreted non-zero covered interest rate differentials as evidence of lack of free capital mobility. In this section we introduce the uncovered interest rate parity condition and show that violations of uncovered interest rate parity can occur even under free capital mobility. We then discuss some empirical studies of uncovered interst rate parity.

8.3.1 Asset Pricing in a 2-Period Small Open Economy

Consider a small open endowment economy with free capital mobility. Assume that, like in the model studied in chapter 4, there is uncertainty about period 2. In period 1, the nominal endowment is equal to Q_1. In period 2, the economy is with probability π in the good state and the endowment is high and equal to Q_2^g and with probability $1-\pi$ in the bad state and the endowment is low and equal to $Q_2^b < Q_2^g$:

$$Q_2 = \begin{cases} Q_2^g & \text{with probability } \pi \\ Q_2^b & \text{with probability } (1-\pi) \end{cases} .$$

Households have access to domestic and foreign currency denominated, nominally risk free, one-period bonds. Let B_1 denote domestic currency bonds purchased in period 1. Domestic currency bonds pay the nominal interest rate i_1 when held from period 1 to period 2. The foreign nominal interest rate is equal to i_1^*. Let B_1^* denote the quantity of foreign currency bonds the domestic household acquires in period 1 and for which the household buys forward cover. That is, in period 1 the household enters into a contract that allows it to convert in period 2 $(1+i_1)B_1^*$ units of foreign currency into domestic currency at the forward exchange rate, F_1. Let $\tilde{B}_1^*$ denote the quantity of foreign currency bonds the domestic household acquires in period 1 but for which it does not acquire forward cover and hence is exposed to some exhchange rate risk.

The variable S_t denotes the domestic currency price of one unit of foreign currency in period t, or the the spot exchange rate in period t. We can then express the period 1 budget constraint of the domestic household as

$$P_1C_1 + B_1 + S_1B^*1_1 + S_1\tilde{B}_1^* = Q_1 \tag{8.7}$$

where P_1 denotes the domestic price level in period 1. Here we have assumed that the household entered period 1 without any asset holdings, $B_0 = 0$.

The budget constraint in the good state in period 2 is

$$P_2^gC_2^g = Q_2^g + (1+i_1)B_1 + F_1(1+i_1^*)B_1^* + S_2^g(1+i_1^*)\tilde{B}_1^*. \tag{8.8}$$

The variable P_2^g denotes the price level in the good state in period 2, C_2^g denotes the level of consumption in the good state in period 2, and S_2^g denotes the spot exchange rate in period 2 in the good state. Similarly, the budget constraint in the bad state in period 2 is given by

$$P_2^bC_2^b = Q_2^b + (1+i_1)B_1 + F_1(1+i_1^*)B_1^* + S_2^b(1+i_1^*)\tilde{B}_1^*. \tag{8.9}$$

The variable P_2^b denotes the price level in the bad state in period 2, C_2^b denotes the level of consumption in the bad state in period 2, and S_2^b denotes the spot exchange rate in the bad state in period 2. Notice that we already imposed the no Ponzi game condition, that households cannot have any debts at the end of period 2 and that in addition they choose not to leave any assets in the last period of life. Formally, we have imposed that $B_2 = B_2^* = \tilde{B}_2^* = 0$.

In period 1, the expected utility of a household is given by

$$\mathcal{U} = U(C_1) + \pi U(C_2^g) + (1 - \pi)U(C_2^b), \tag{8.10}$$

where $U(.)$ is an increasing and concave period utility function.

The household's maximization problem hence consist in choosing C_1, C_1^g, C_1^b, B_1, B_1^*, and $\tilde{B}_1^*$ to maximize expected utility, (8.10), subject to the period-by-period budget constraints, (8.7), (8.8), and (8.9), taking as given prices, P_1, P_2^g, P_2^b, spot exchange rates, S_1, S_2^g, S_2^b, the forward rate, F_1, interest rates, i_1 and i_1^*, and the endowments, Q_1, Q_2^g, Q_2^b. This problem looks complicated, we must choose 6 variables to maximize utility subject to three constraints. Following the strategy used in chapter 4, we solve the period-1 budget constraint for C_1, the period-2 good state budget constraint for C_2^g, and the period-2 bad state budget constraint for C_2b. Then we can use the three resulting expresssions to eliminate C_1, C_2^g and C_2^b from the utility function and we obtain a problem for choosing B_1, B_1^*, and $\tilde{B}_1^*$ to maximize utility.

Solving the budget constraint (8.7) for C_1, we can express period-1 consumption as a function of bond holdings as follwos

$$C_1(B_1, B_1^*, \tilde{B}_1^*) = \frac{Q_1 - B_1 - S_1 B_1^* - S_1 B_1^*}{P_1}.$$

Similarly, solving the period-2, good state, budget constraint for C_2^g yields:

$$C_2^g(B_1, B_1^*, \tilde{B}_1^*) = \frac{Q_2^g + (1+i)B_1 + (1+i^*)\left(F_1 B_1^* + S_2^g B_1^*\right)}{P_2^g}$$

and solving the period-2, bad state, budget constraint for C_2^b yields

$$C_2^b(B_1, B_1^*, \tilde{B}_1^*) = \frac{Q_2^b + (1+i)B_1 + (1+i^*)\left(F_1 B_1^* + S_2^b B_1^*\right)}{P_2^b}$$

We can then use these three expressions to eliminate consumption of the utility function to obtain

$$\mathcal{U}(B_1, B_1^*, \tilde{B}_1^*) = U(C_1(B_1, B_1^*, \tilde{B}_1^*)) + \pi\, U(C_2^g(B_1, B_1^*, \tilde{B}_1^*)) + (1-\pi) U(C_2^b(B_1, B_1^*, \tilde{B}_1^*)).$$

Now take the first-order condition with respect to B_1 and equate it to zero

$$0 = \frac{\partial \mathcal{U}}{\partial B_1} \quad \Rightarrow \quad U'(C_1)\frac{1}{P_1} = \pi U'(C_2^g)\frac{1+i}{P_2^g} + (1-\pi)U'(C_2^b)\frac{1+i}{P_2^b}$$

The left-hand-side of the latter expression indicates the marginal utility of one unit of domestic currency in period 1. One unit of domestic currency buys $1/P_1$ units of goods, which in turn, provide marginal utility in the amount of $U'(C_1)$. The right-hand-side of the latter expression also indicates the marginal utility of one unit of currency in period 1, but in this case, the one unit of currency is used to buy a domestic bond, this bond returns $(1+i)$ units of currency in period 2 regardless of the state of the economy. In the good state, this buys $1/P_2^g$ units of goods, which each provide $U'(C_2^g)$ units of marginal utility and in the bad state this buys $1/P_2^b$ units of goods, which each provide $U'(C_2^b)$ units of marginal utility. The right hand side thus gives the expected marginal utility of investing one unit of domestic currency in period 1 in the domestic bond and consuming the proceeds in period 2. At the optimum, the expected utility of investing one unit of domestic currency in the domestic currency or converting it into consumption goods already in period 1 must generate the same level of marginal utility.

Rewrite this first-order condition as

$$1 = (1+i)\left[\pi \frac{U'(C_2^g)}{U'(C_1)} \frac{P_1}{P_2^g} + (1-\pi) \frac{U'(C_2^b)}{U'(C_1)} \frac{P_1}{P_2^b}\right]$$

Letting E_1 denote the expectations operator conditional on information available in period 1, we have:

$$1 = (1+i)E_1 \left\{ \frac{U'(C_2)}{U'(C_1)} \frac{P_1}{P_2} \right\}$$

Let $M_2 \equiv \left\{ \frac{U'(C_2)}{U'(C_1)} \frac{P_1}{P_2} \right\}$ denote the nominal marginal rate of substitution between period 2 and period 1, to arrive at the following asset pricing condition:

$$1 = (1+i)E_1 \{M_2\} \tag{8.11}$$

And the first-order-condition with respect to foregin bonds B^*1 for which the household buys forward cover is

$$0 = \frac{\partial \mathcal{U}}{\partial B_1^*} \quad \Rightarrow \quad U'(C_1)\frac{S_1}{P_1} = \pi(1+i^*)U'(C_2^g)\frac{F_1}{P_2^g} + (1-\pi)(1+i^*)U'(C_2^b)\frac{F_1}{P_2^b}$$

Rewrite this expression as

$$1 = (1+i^*)\frac{F_1}{S_1}\left[\pi\frac{U'(C_2^g)}{U'(C_1)}\frac{P_1}{P_2^g} + (1-\pi)\frac{U'(C_2^b)}{U'(C_1)}\frac{P_1}{P_2^b}\right]$$

Using the expectations operator notation we have

$$1 = (1+i^*)\frac{F_1}{S_1}E_1\left\{\frac{U'(C_2)}{U'(C_1)}\frac{P_1}{P_2}\right\}$$

or

$$1 = (1+i^*)\frac{F_1}{S_1}E_1\left\{M_2\right\} \tag{8.12}$$

Combining (8.11) and (8.12) we obtain:

$$\boxed{(1+i) = (1+i^*)\tfrac{F_1}{S_1}} \tag{8.13}$$

which is the **covered interest rate parity condition**, we had set out to derive.

To recap, we have shown that under free capital mobility, covered interest rate parity must hold.

Uncovered interest rate parity holds when the rate of return on the domestic bond is equal to the expected rate of return on a foreign bond.

Specifically, we say that uncovered interest rate parity holds if

$$(1+i) = (1+i^*)E_1\left\{\frac{S_2}{S_1}\right\}$$

Comparing the uncovered interst rate parity condition to the covered interst rate parity condition, it follows that uncovered interst rate parity holds if

$$F_1 = E_1 S_2$$

that is, if the forward rate, F_1, is equal to the expected future spot rate, $E_1 S_2$.

Does uncovered interest rate parity need to hold, or equivalently, must the forward rate equal the expected future spot rate, when international capital markets are fully integrated? To answer this question in the context of our model consider the optimal choice of risky foreign bonds, that is, of foreign currency bonds whose exchange rate risk the household does not insure through the purchase of forward exchange rate contracts. The first-order optimality condition of expected utility with respect to $\tilde{B}_1^*$ must be zero, or,

$$0 = \frac{\partial \mathcal{U}}{\partial \tilde{B}_1^*} \quad \Rightarrow \quad U'(C_1)\frac{S_1}{P_1} = \pi(1+i^*)U'(C_2^g)\frac{S_2^g}{P_2^g} + (1-\pi)(1+i^*)U'(C_2^b)\frac{S_2^b}{P_2^b}$$

Rewrite this expression as

$$1 = (1+i^*)\left[\pi\frac{S_2^g}{S_1}\frac{U'(C_2^g)}{U'(C_1)}\frac{P_1}{P_2^g} + (1-\pi)\frac{S_2^b}{S_1}\frac{U'(C_2^b)}{U'(C_1)}\frac{P_1}{P_2^b}\right]$$

Using the expectations operator notation we have:

$$1 = (1+i^*)E_1\left\{\left(\frac{S_2}{S_1}\right)\left(\frac{U'(C_2)}{U'(C_1)}\frac{P_1}{P_2}\right)\right\} = (1+i^*)E_1\left\{\left(\frac{S_2}{S_1}\right)M_2\right\} \quad (8.14)$$

Combining the asset pricing equations (8.14) and (8.12) we obtain

$$F_1 E_1 M_2 = E_1 S_2 M_2$$

But this expression does in general not imply that the forward rate, F_1, is equal to the expected future spot rate, S_2. That is, it does not follow from here that

$$F_1 = E_1 S_2$$

Hence, under free capital mobility, uncovered interest rate parity in general fails to hold. It follows that if we observe deviations from uncovered interest parity, we cannot conclude that there is incomplete capital market integration. This was the second result we had set out to show.

While uncovered interest rate parity does not hold in general, there are conditions under which it does indeed obtain. In what follows we show that if the pricing kernel, M_2, is uncorrelated with the depreciation rate of the domestic currency, $S_2/S1$, then uncovered interest rate parity should hold.

Recall that for any pair of random variables a and b

$$\begin{aligned} cov(a,b) &= E\left(a - E(a)\right)\left(b - E(b)\right) \\ &= E\left(ab\right) - E(a)E(b) \end{aligned}$$

or

$$E\,(ab) = cov(a, b) + E(a)E(b),$$

where *cov* denotes the covariance between a and b.

We then can express $E_1 M_2 (S_2/S_1)$ as

$$E\left(\frac{S_2}{S_1} M_2\right) = cov\left(\frac{S_2}{S_1}, M_2\right) + E\left(\frac{S_2}{S_1}\right) E(M_2)$$

and rewrite (8.14) as

$$1 = (1 + i^*)\left[cov\left(\frac{S_2}{S_1}, M_2\right) + E\left(\frac{S_2}{S_1}\right) E(M_2)\right]$$

If the depreciation rate, S_2/S_1, is uncorrelated with the pricing kernel, M_2, that is, if

$$cov\left(\frac{S_2}{S_1}, M_2\right) = 0$$

thenequation (8.14) becomes

$$1 = (1 + i^*)E_1\left(\frac{S_2}{S_1}\right) E_1(M_2)$$

Combining this expression with equation (8.12) to obtain

$$F_1 = E_1 S_1 \tag{8.15}$$

It follows that if the depreciation rate is uncorrelated with the pricing kernel, M_2, then the forward rate equals the expected future spot rate. And further if we combine the above expression with (8.11) we have

$$\boxed{(1+i) = (1+i^*)E_1\left\{\frac{S_2}{S_1}\right\}}$$

or **Uncovered Interest Rate Parity** holds.

We have therefore shown that while uncovered interest rate parity need not hold in general there are special circumstances in which it might hold.

Next we look at some empirical evidence on uncovered interst rate differentials. We will see that in actual data uncoverend interest rate differential are not zero, that is, we will present empirical evidence that uncovered interest rate parity fails.

8.3.2 The Forward Premium Puzzle

When a currency is 'more expensive' in the forward market than in the spot market, that is, when

$$F_t < S_t,$$

then we say that the domestic currency is at a premium in the forward market. (And the foreign currency is at a discount in the forward market.) Rearraning the covered interest rate parity condition we have

$$\frac{(1+i_t)}{(1+i_t^*} = \frac{F_t}{S_t}.$$

Hence, it must be the case that when the domestic currency is trading at a premium in the forward market, $F_t/S_t < 1$, the domestic interest rate must be lower than the foreign interest rate. It follows that low interest rate currencies are at a premium in the forward market.

Suppose for the moment that uncovered interest rate parity held, that is, that $1+i_t = (1+i_t^*)S_{t+1}/S_t$. If UIRP holds, then low interest rate currencies are expected to appreciate, that is, S_{t+1} is expected to fall. In particular, the domestic currency would be expected to appreciate by exactly the interest rate differential.

This is a testable prediction. One can look at time series evidence on relative interest rates, $(1+i_t)/(1+i_t^*)$ and compare it to realized appreciations, S_{t+1}/S_t. If UIRP holds, then on average the interest rate differential has to equal the average appreciation. In actual data, however, the realized appreciation is on average smaller than the forward premium. Letting f_t denote the natural logarithm of the forward rate and s_t the natural logarithm of the spot exchange rate can express the forward premium as

$$\text{forward premium} = f_t - s_t.$$

Using that fact that for x small, $\ln(1+x) \approx x$ we can express the covered interst rate parity condition as

$$i_t - i_t^* = f_t - s_t$$

and the uncovered interest rate parity condition as

$$i_t - i_t^* = E(s_{t+1} - s_t)$$

The finding of empirical studies is that the interest rate differential, $i_t - i_t^*$

systematically exceeds the appreaciation of the domestic currency, $s_{t+1} - s_t$.

8.3.3 Carry Trade

This empirical regularity suggests the following investment strategy. Borrow the low interst rate currency, invest in the high interest rate currency, and do not hedge in the forward market. This investment strategy is known as carry trade and is widely used by practicioners.

Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006) document returns to carry trade for the pound sterling over the period 1976:1 to 2005:12. In their study i_t corresponds to either the 1-month or the 3-month pound sterling interest rate.

Burnside et al. use monthly data from 1976:1 to 2005:12. In their study the domestic country is the UK, so i_t is either the 1-month or the 3-month pound sterling interest rate. They collect data on 1-month and 3-month forward exchange rates of the pound, F_t, on spot exchange rates, S_t, and on foreign 1-month and 3-month interest rates. i_t^*. Then they compute the average payoff from carry trade — taking into account that there are some transaction costs. And they find that the average return to carry trade is **positive**. The following table is taken from their paper.

Observations on Table 4 of Burnside et al. (2006)

1. Consider the case with corrections for transactions cost, columns 5, 6, and 7 of the table. The average return to carry trade for an equally-weighted portfolio of the 10 currencies considered is 0.0029 per unit of currency invested for one month.

TABLE 4

Payoffs to the Carry Trade Strategies 76:01-05:12

	No Transactions Costs			With Transactions Costs		
	Mean	Standard Deviation	Sharpe Ratio	Mean	Standard Deviation	Sharpe Ratio
Belgium*	0.0044	0.028	0.157	0.0029	0.021	0.140
	(0.0019)	(0.002)	(0.068)	(0.0015)	(0.002)	(0.072)
Canada	0.0053	0.032	0.169	0.0042	0.026	0.161
	(0.0018)	(0.002)	(0.059)	(0.0014)	(0.002)	(0.055)
France*	0.0054	0.027	0.201	0.0031	0.023	0.134
	(0.0016)	(0.002)	(0.060)	(0.0015)	(0.002)	(0.066)
Germany*	0.0011	0.028	0.038	0.0014	0.024	0.060
	(0.0018)	(0.002)	(0.066)	(0.0016)	(0.002)	(0.068)
Italy*	0.0029	0.028	0.105	0.0024	0.024	0.102
	(0.0017)	(0.002)	(0.058)	(0.0014)	(0.002)	(0.056)
Japan†	0.0022	0.036	0.061	0.0017	0.034	0.049
	(0.0022)	(0.003)	(0.063)	(0.0020)	(0.003)	(0.060)
Netherlands*	0.0024	0.028	0.087	0.0014	0.023	0.062
	(0.0018)	(0.002)	(0.068)	(0.0015)	(0.002)	(0.067)
Switzerland	0.0019	0.030	0.063	0.0008	0.027	0.028
	(0.0017)	(0.002)	(0.060)	(0.0015)	(0.002)	(0.057)
USA	0.0039	0.031	0.124	0.0030	0.029	0.103
	(0.0017)	(0.002)	(0.058)	(0.0016)	(0.002)	(0.059)
Euro‡	0.0014	0.021	0.066	0.0024	0.016	0.153
	(0.0017)	(0.002)	(0.083)	(0.0013)	(0.002)	(0.090)
Average	0.0031	0.029	0.107	0.0023	0.025	0.099
Equally-weighted portfolio	0.0031	0.017	0.183	0.0029	0.020	0.145
	(0.0009)	(0.001)	(0.061)	(0.0011)	(0.001)	(0.057)

* Euro legacy currencies available 76:1-98:12
† Japanese yen available 78:7-05:12
‡ Euro available 99:1-05:12
Notes: Other currencies and the equally-weighted portfolio are available for 76:1-05:12. Standard errors in parentheses.

Source: THE RETURNS TO CURRENCY SPECULATION Burnside, Eichenbaum, Kleshchelski, and Rebelo, NBER WP 12489, August 2006.

2. To generate substantial profits, speculators must wager very large sums of money. For example, suppose $y = 1,000,000,000$, that is, you invest one billion pounds in carry trade, then after one month the carry trade had, over the sample period, an average payout of 2.9 million pounds per month.

3. The fact that the average payoff from carry trade is non-zero implies that UIRP fails empirically.

4. Columns 4 and 7 of the table report the Sharpe Ratio, which is defined as

$$\text{Sharpe ratio } = \frac{\text{mean(payoff)}}{\text{std of payoff}}$$

   The Sharpe ratio is a measure of risk. The higher the Sharpe ratio, the higher the risk adjusted return. For comparision, note that the Sharpe ratio of investing in the S&P 500 index over the sample period was 0.14, which is comparable to the Sharpe ratio of the carry trade.

   But remember the carry trade is an arbitrage, so you don't need to have any capital to execute it.

5. Q: Suppose a speculator wants to generate a payoff of 1 million pounds on average per year. How large a carry trade must he engage in? A: He needs GBP 28.3 million each month.

6. Burnside et al. also compute covered interest rate differentials (not shown) and find that CIRP parity holds in their sample.

Carry Trade returns are thought to have crash risk. The Economist article refers to carry trade returns as "picking up nickels in front of steamrollers."

Example: large surprise appreciation of the Japanese Yen against the U.S. dollar on October 6-8, 1998. The Yen appreciated by 14 percent (or equivalently the U.S. dollar depreciated by 14 percent).

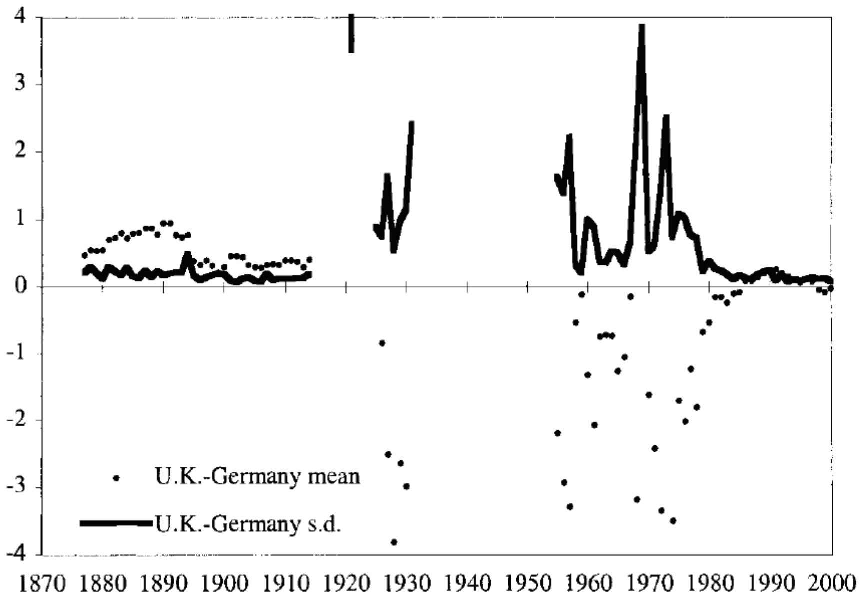
Suppose that you were a carry trader with 1 billion dollars short in Yen and long in U.S. dollars. The payoff of that carry trade in the span of 2 days was -140 million dollars — that is, the steamroller caught up with the carry trader.

8.4 Exercises

1. [**International Capital Mobility: 1870 to 2000**] Forward exchange contracts of the kind common after 1920 were not prevalent before then. However, there was another widely traded instrument, called the long bill of exchange. Long bills could be used to cover the exchange risk that might otherwise be involved in interest-rate arbitrage. Let b_t denote the long bill rate, which is defined as the date-t dollar price in New York of £1 deliverable in London after ninety days. (Note that b_t is paid 90 days prior to the date of delivery of the £.) Let i_t^* denote the 90-day deposit rate in London, i_t the 90-day deposit rate in New York, and S_t the spot exchange rate, that is, the dollar price of one British pound.

   (a) Suppose you had time series data for b_t, S_t, i_t and i_t^*. How can you construct a test of free capital mobility between the United States and Great Britain in the period prior to 1920.

   (b) The figure shows annualized covered interest rate differentials between Germany and the United Kingdom over the period 1870 to 2000. What can you deduce from the figure about the degree of international capital mobility between these two countries over time.



Source: This is figure 3.5 of Maurice Obstfeld and Alan M. Taylor, 'Globalization and Capital Markets,' in "Globalization in Historical Perspective," Michael D. Bordo, Alan M. Taylor and Jeffrey G. Williamson, editors, University of Chicago Press, January 2003.

Chapter 9

Determinants of the Real Exchange Rate

You might have noticed that sometimes Europe seems much cheaper than the United States and sometimes it is the other way around. In the first case, we have incentives to travel abroad and import goods and services. In the second case, European tourists come in larger numbers and we have an easier time exporting goods and services to Europe. What factors determine how cheap or expensive a country is relative to others? Addressing this question is the focus of this chapter and the next.

9.1 The Law of One Price

When a good costs the same abroad and at home, when prices are expressed in the same currency, we say that the Law of One Price holds. Let P denote the domestic-currency price of a particular good in the domestic country,

P^* the foreign-currency price of the same good in the foreign country, and S the nominal exchange rate, defined as the domestic-currency price of one unit of foreign currency. The Law of One Price holds if

$$P = SP^*.$$

The good is more expensive in the foreign economy if $SP^* > P$, and less expensive if $SP^* < P$.

Let's begin by studying whether the Law of One Price holds for McDonald's Big Mac sandwich. This is handy to do because The Economist Magazine has been collecting data on the price of Big Macs around the world since 1986. The Big Mac is also a good example because it is made pretty much the same way all over the world, so we are sure we are comparing the price of the same good across countries. A third advantage of considering the Big Mac is that most of you (if not all) have eaten one at some point in your lives, which makes it a product one can easily relate to. Thus let P^{BigMac} denote the dollar price of a Big Mac in the United States and $P^{\text{BigMac}*}$ the foreign-currency price of a Big Mac in the foreign country. Then we can construct a measure of how many Big Macs one can buy abroad for one Big Mac in the United States. This measure is called the Big-Mac real exchange rate, and we denote it by e^{BigMac}. Formally, e^{BigMac} is given by

$$e^{\text{BigMac}} = \frac{SP^{\text{BigMac}*}}{P^{\text{BigMac}}}.$$

If $e^{\text{BigMac}} > 1$, then the Big Mac is more expensive abroad, that is, if you exchanged the dollar value of one Big Mac in the U.S. into foreign currency,

Table 9.1: Big Mac Prices around the World, 2014

Country	$SP^{\text{BigMac}*}$ (in U.S. dollars)	e^{BigMac}
Norway	7.80	1.69
Switzerland	7.14	1.54
Brazil	5.25	1.13
Canada	5.01	1.08
Euro area	4.96	1.07
Britain	4.63	1.00
United States	**4.62**	**1**
Australia	4.47	0.97
Turkey	3.76	0.81
Japan	2.97	0.64
China	2.74	0.59
Russia	2.62	0.57
Egypt	2.43	0.53
Indonesia	2.30	0.50
South Africa	2.16	0.47

Source: The Economist Magazine, January 25th, 2014, page 63.

you would not have enough money to buy one Big Mac abroad. If the law of one price holds for Big Macs, then

$$e^{\text{BigMac}} = 1.$$

Table 9.1 presents Big Mac real exchange rates for 15 countries measured in January 2014. The table shows that for the Big Mac the law of one price clearly fails. In Norway, the most expensive country in our sample, a Big Mac sells for the equivalent of \$7.80 whereas in the United States it sells for \$4.62. Thus, one Big Mac in Norway costs the same as 1.69 Big Macs in the United States. By contrast, Big Macs in South Africa are much cheaper than in the United States, selling for just \$2.16, so that for the price of one Big Mac in the U.S. one can buy 2.14 Big Macs in South Africa.

9.2 Purchasing Power Parity

Purchasing power parity (PPP) is the generalization of the idea of the law of one price. Suppose the law of one price holds for a basket of goods. Accordingly, we now let P denote the domestic currency price of a basket of goods, P^* the foreign currency price of a basket of goods, and S continues to denote the domestic currency price of one unit of foreign currency. Then we define the real exchange rate, e, as

$$e = \frac{SP^*}{P}.$$

The real exchange rate e indicates the relative price of a consumption basket in the foreign country in terms of consumption baskets in the home country. When the purchasing power of the domestic currency is equal to the purchasing power of the foreign currency, that is, when $P = SP^*$, we say that absolute Purchasing Power Parity (PPP) holds, or

$$e = 1.$$

Does Absolute PPP hold?

How can we test absolute PPP? We would need data on P and P^*. This is much trickier than it might seem at first glance. Statistical agencies typically produce a price index, such as the consumer price index. And one might think the obvious empirical counterpart of P is the CPI index. But the problem is that the CPI is an index and not the actual level of prices. Price indices typically are arbitrarily normalized to be 100 in a base year. So they provide information about how the price of a basket of goods changes over time, but not about the level of the price of such basket. If the price index in Europe is higher than in the U.S., you cannot tell which country is more expensive Europe or the United States. To determine the purchasing power of the U.S. dollar, you need information on the actual price level rather than on the level of the price index. But, again, price indices are extremely useful to study how the real exchange rate changes over time. We will use this type of information in the next section.

The World Bank's International Comparison Program (ICP) produces data on price levels.[1] The most recent ICP benchmark contains price level data for 146 countries, including 100 developing and emerging economies and 46 advanced economies in 2005 based on prices of more than 1000 individual goods. This program produces a measure of the level of the real exchange rate and hence allows testing of absolute PPP. Table 9.2 shows the level of the real exchange rate of a number of developing and advanced economies relative to the United States.

The table shows that goods that cost \$26 in Ethiopia cost \$100 in the United States, that is, prices are by a factor of 4 lower in Ethiopia. Or consider India, goods that cost \$33 in India would cost \$100 in the United States. Hence the price level in the United States is three times the (ex-

[1]See the report 'Global Purchasing Power Parities and Real Expenditures,' 2005 International Comparison Program, The World Bank, 2008.

Table 9.2: Absolute PPP fails

Country	$100e = 100\frac{SP^*}{P^{US}}$
United States	100
Ethiopia	26
Bangladesh	35
India	33
Pakistan	32
China	42
Germany	111
Sweden	124
Switzerland	140
Japan	118

Source: Global Purchasing Power Parities and Real Expenditures, 2005 International Comparison Program, The World Bank, 2008.

change rate adjusted) price level in India. Not all countries are cheaper than the United States. Prices are quite a bit higher in Switzerland than in the United States. Exercise 1 at the end of this chapter asks you to compare ICP real exchange rates for the year 2005 with Big Mac Real Exchange rates for 2005, to see to which extend the Big Mac is a good indicator of how expensive or cheap a country is on the whole.

Relative PPP

Many studies on purchasing power parity focus on changes in the real exchange rate over time. The great advantage of this is that one does not need actual price level data and can instead work with widely available consumer price index data. We say that that relative PPP holds if

$$\Delta e = 0.$$

Under relative PPP the same basket of goods might not fetch the same price in two countries but if the price of that basket of goods changes in one country in also changes in the other country. Clearly, if the real exchange rate appreciates, e falls, then we can say that the domestic country became more expensive relative to the foreign country.

Taylor and Taylor (2005) study whether relative PPP holds over the short and the long run. For testing long-run relative PPP, they consider changes in the dollar-pound real exchange rate, defined as,

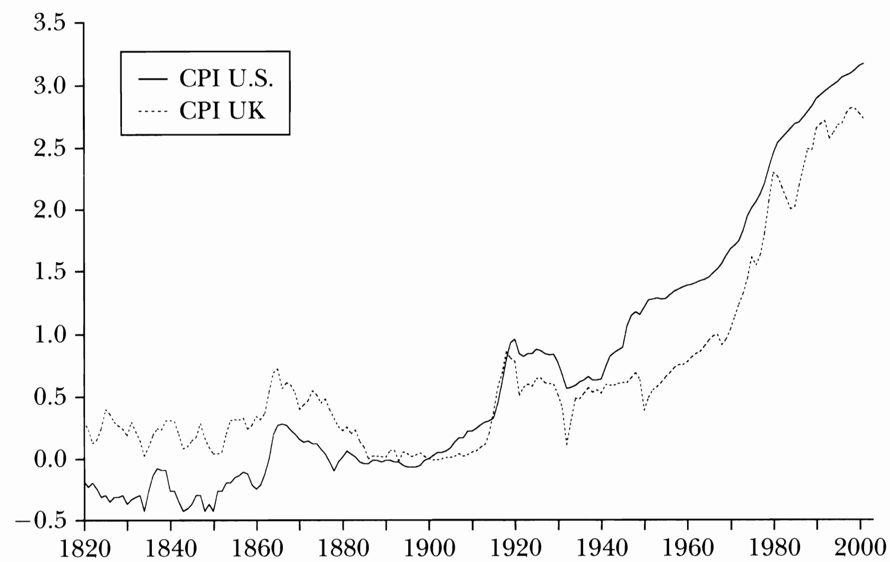
$$e^{\$/£} = \frac{S^{\$/£} P^{UK}}{P^{US}},$$

where $S^{\$/£}$ is the dollar-pound nominal exchange rate, defined as the dollar price of one pound, P^{UK} is the price index in the U.K., and P^{US} is the price index in the United States. Figure 9.1 shows with a solid line the U.S. consumer price index and with a broken line the U.K. consumer price index expressed in U.S. dollars over the period 1820-2001 using a log scale. The vertical difference between the two lines is a measure of the dollar-pound real exchange rate. Figure 9.1 shows that over the long run P^{US} and $S^{\$/£}P^{UK}$ move in tandem. This empirical fact suggests that relative PPP is a useful approximation to actual real exchange rate behavior over long horizons.

If, as suggested by figure 9.1, relative PPP holds over the long run, then it must be the case that, on average,

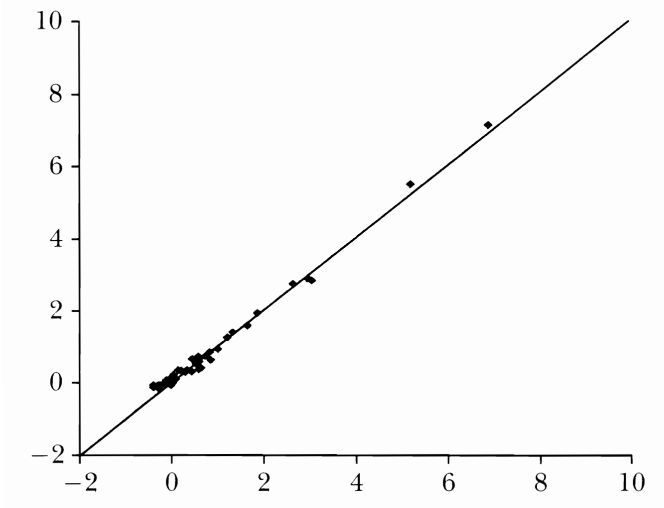
$$\%\Delta P^* - \%\Delta P = -\%\Delta S, \tag{9.1}$$

Figure 9.1: Dollar-Sterling PPP Over Two Centuries



Note: The figure shows U.S. and U.K. consumer price indices expressed in U.S. dollar terms over the period 1820-2001 using a log scale with a base of 1900=0. Source: Alan M. Taylor and Mark P. Taylor, "The Purchasing Power Parity Debate," *Journal of Economic Perspectives* 18, Fall 2004, 135-158.

Figure 9.2: Consumer Price Inflation Relative to the U.S. Versus Dollar Exchange Rate Depreciation, 29-Year Average, 1970-1998



Note: The figure shows countries' cumulative inflation rate differentials against the United States in percent (vertical axis) plotted against their cumulative depreciation rates against the U.S. dollar in percent (horizontal axis). The sample includes data from 20 industrialized countries and 26 developing countries. Source: Alan M. Taylor and Mark P. Taylor, "The Purchasing Power Parity Debate," *Journal of Economic Perspectives* 18, Fall 2004, 135-158.

where $\%\Delta P^*$, $\%\Delta P$, and $-\%\Delta S$, denote, respectively, the percentage change in the foreign price level (or the rate of foreign inflation), the percent change in the domestic price level (or the rate of domestic inflation), and the percent change in the nominal exchange rate (or rate of nominal exchange rate depreciation of the foreign currency).

Figure 9.2 shows a scatterplot of average inflation differentials relative to the U.S. $\%\Delta P^* - \%\Delta P^{US}$, against rates of dollar exchange rate depreciation $-\%\Delta S$, over the period 1970 and 1998 for 20 developed countries and 26 developing countries. Equation (9.1) states that if relative PPP holds over the long run, then the points on the scatterplot should lie on a line with slope equal to 1 and an intercept of zero. The figure shows that this relation is quite accurate for both high- and low-inflation countries: countries with high average exchange rate depreciations were countries that experienced high average inflation-rate differentials vis-à-vis the United States, and countries whose currency did not depreciate much vis-à-vis the U.S. dollar tended to have low average inflation differentials.

9.3 Deviations from PPP due to Nontradables

In the two-period model we developed in chapters 3 and 5, there is a single traded good. Thus, under the maintained assumption of free international trade, relative purchasing power parity obtained, that is, $\Delta e = 0$.

As we have just seen, this prediction of our model obtains in the long run but not in the medium to short runs. Why does our model fail to predict short- to medium-run deviations from PPP? One reason is that in reality, contrary to what is assumed in the model, not all goods are tradable. Examples of nontraded goods are services, such as haircuts, restaurant meals, housing, some health services, and some educational services. For these goods transport costs are so large relative to the production cost that they cannot be traded internationally at a profit. For instance, few will fly to from the U.S. to India just to take advantage of a 15-dollar price differential in hair cuts. Goods and services with these characteristics are called nontradables.

In general, nontradables make up a significant share of a country's output, typically above 50 percent. The existence of nontradables allows for systematic violations of PPP. To see this, note that the price index P is an average of all prices in the economy. Consequently, it includes both the prices of nontradables and the prices of tradables. But the prices of nontradables are determined entirely by domestic factors, so one should not expect the law of one price to hold for this type of goods.

To see how changes in the price of nontradables can cause changes in the real exchange rate, let P_T and P_N denote the domestic prices of tradables and nontradables, respectively, and let P_T^* and P_N^* denote the corresponding foreign prices. For traded goods the law of one price should hold, that is,

$$P_T = EP_T{}^*,$$

but for nontraded goods it need not

$$P_N \neq EP_N{}^*.$$

Suppose the price level, P, is constructed as some average of the price of tradables and nontradables. We can then write

$$P = \phi(P_T, P_N),$$

where ϕ is increasing in P_T and P_N and homogeneous of degree one.[2] For instance, if P is a geometric average of P_T and P_N, then $\phi(P_T, P_N) =$

[2]A function $f(x, y)$ is homogeneous of degree one if $f(\lambda x, \lambda y) = \lambda f(x, y/)$.

$(P_T)^\alpha (P_N)^{1-\alpha}$, with $\alpha \in (0,1)$. If P were a simple average of P_T and P_N, then we would have that $\phi(P_T, P_N) = (P_T + P_N)/2$. The assumption that $\phi(\cdot,\cdot)$ is homogeneous of degree one ensures that, if all individual prices increase by, say, 5%, then P also increases by 5%. Assume that the price level in the foreign country is also constructed as some average of the prices of tradables and nontradables, that is

$$P^* = \phi(P_T^*, P_N^*).$$

We can then write the real exchange rate, e, as

$$\begin{aligned} e &= \frac{EP^*}{P} \\ &= \frac{E\phi(P_T^*, P_N^*)}{\phi(P_T, P_N)} \\ &= \frac{EP_T^*\phi(1, P_N^*/P_T^*)}{P_T\phi(1, P_N/P_T)} \\ &= \frac{\phi(1, P_N^*/P_T^*)}{\phi(1, P_N/P_T)}. \end{aligned} \tag{9.2}$$

So the real exchange rate should depend on the ratio of nontraded to traded prices in both countries. The real exchange rate is greater than one (or the price of the foreign consumption basket is higher than the price of the domestic consumption basket) if the relative price of nontradables in terms of tradables is higher in the foreign country than domestically. Formally,

$$e > 1 \text{ if } \frac{P_N^*}{P_T^*} > \frac{P_N}{P_T}.$$

It is straightforward to see from this inequality that e can increase over time if the price ratio on the left-hand side increases over time more than the one on the right hand side.

When considering a particular country pair, it is useful to define a bilateral real exchange rate. For example, the dollar-yen real exchange rate is given by

$$e^{\$/¥} = \frac{E^{\$/¥} P^{Japan}}{P^{U.S.}} = \frac{\text{Price of Japanese goods basket}}{\text{Price of US goods basket}}.$$

Suppose $e^{\$/¥}$ increases, then the price of the Japanese goods basket in terms of the U.S. goods basket increases. In this case, we say that the dollar real exchange rate vis-à-vis the yen depreciated, because it takes now more U.S. goods baskets to purchase one Japanese goods basket.

At this point, a word of caution about semantics is in order. Economists use the term real exchange rate loosely. The term real exchange rate is sometimes used to refer to EP^*/P and sometimes to refer simply to P_T/P_N. A real exchange rate appreciation means that either EP^*/P falls or that P_T/P_N falls, depending on the concept of real exchange rate being used. Similarly, a real exchange rate depreciation means that either EP^*/P goes up or that P_T/P_N goes up.

Next we turn to an analysis of the determinants of real exchange rates. We begin by studying a theory that explains medium-run variations in bilateral real exchange rates. What do we mean by medium run in this context? Take another look at figure 9.1. The figure shows that over a period of 180 years prices in the United States and the United Kingdom expressed in the same currency changed by about the same magnitude. However, they deviated significantly on a period-by-period basis. and, more importantly, these deviations were fairly persistent. For instance, during the 1980s the U.S. price level grew faster than the U.K. counterpart measured in the same currency. That is, a representative basket of goods became relatively more expensive in the United States than in the United Kingdom, or the dollar-pound exchange rate experienced a prolonged real appreciation. The theory that follows explains these medium-term deviations in PPP as resulting from differences across countries in the productivity of the tradable sector relative to the productivity of the nontradable sector.

9.4 Productivity Differentials and Real Exchange Rates: The Balassa-Samuelson Model

According to the Balassa-Samuelson model deviations from PPP are due to cross-country differentials in the productivity of technology to produce traded and nontraded goods. In this section, we study a simple model that captures the Balassa-Samuelson result.

Suppose a country produces 2 kinds of goods, traded goods, Q_T, and nontraded goods, Q_N. Both goods are produced with a linear production technology that takes labor as the only factor input. However, labor productivity varies across sectors. Specifically, assume that output in the traded and nontraded sectors are, respectively, given by

$$Q_T = a_T L_T \tag{9.3}$$

and

$$Q_N = a_N L_N, \tag{9.4}$$

where L_T and L_N denote labor input in the traded and nontraded sectors. Labor productivity is defined as output per unit of labor. Given the linear production technologies, we have that labor productivity in the traded sector is a_T and in the nontraded sector is a_N.[3]

In the traded sector, a firm's profit is given by the difference between revenues from sales of traded goods, $P_T Q_T$, and total cost of production, wL_T, where w denotes the wage rate per worker. That is,

$$\text{profits in the traded sector} = P_T Q_T - wL_T.$$

Similarly, in the nontraded sector we have

$$\text{profits in the nontraded sector} = P_N Q_N - wL_N.$$

We assume that there is perfect competition in both sectors and that there are no restrictions on entry of new firms. This means that as long as profits are positive new firms will have incentives to enter, driving prices down. Therefore, in equilibrium, prices and wages must be such that profits are

[3] There are two concepts of labor productivity: average and marginal labor productivity. Average labor productivity is defined as output per worker, Q/L. Marginal labor productivity is defined as the increase in output resulting from a unit increase in labor input, holding constant all other inputs. More formally, marginal labor productivity is given by the partial derivative of output with respect to labor, $\partial Q/\partial L$. For the linear technologies given in (9.3) and (9.4), average and marginal labor productivities are the same.

zero in both sectors,

$$P_T Q_T = w L_T$$

and

$$P_N Q_N = w L_N.$$

Using the production functions (9.3) and (9.4) to eliminate Q_T and Q_N from the above two expressions, the zero-profit conditions imply

$$P_T a_T = w$$

and

$$P_N a_N = w.$$

Combining these two expressions to eliminate w yields

$$\frac{P_T}{P_N} = \frac{a_N}{a_T}. \tag{9.5}$$

This expression says that the relative price of traded to nontraded goods is equal to the ratio of labor productivity in the nontraded sector to that in the traded sector. To understand the intuition behind this condition suppose that a_N is greater than a_T. This means that one unit of labor produces more units of nontraded goods than of traded goods. Therefore, producing 1 unit of nontraded goods costs less than producing 1 unit of traded goods, and as a result nontraded goods should be cheaper than traded goods ($P_N/P_T < 1$). According to equation (9.5), a period in which labor productivity in the nontraded sector is growing faster than labor productivity in the traded sector will be associated with real exchange rate depreciation (i.e., with P_T/P_N rising).

Is the implication of the Balassa-Samuelson model that the relative price of nontradable goods in terms of tradable goods is increasing in the productivity differential between the traded and nontraded sectors borne out in the data? Figure 9.3 plots the averages of the annual percentage change in P_N/P_T (vertical axis) against the average annual percentage change in a_T/a_N (horizontal axis) over the period 1970-1985 for 14 OECD countries. According to the Balassa-Samuelson model, all observations should line up on the 45-degree line. This is not quite the case. Yet, the data indicate a strong positive relation between difference in total factor productivity and changes in relative prices.

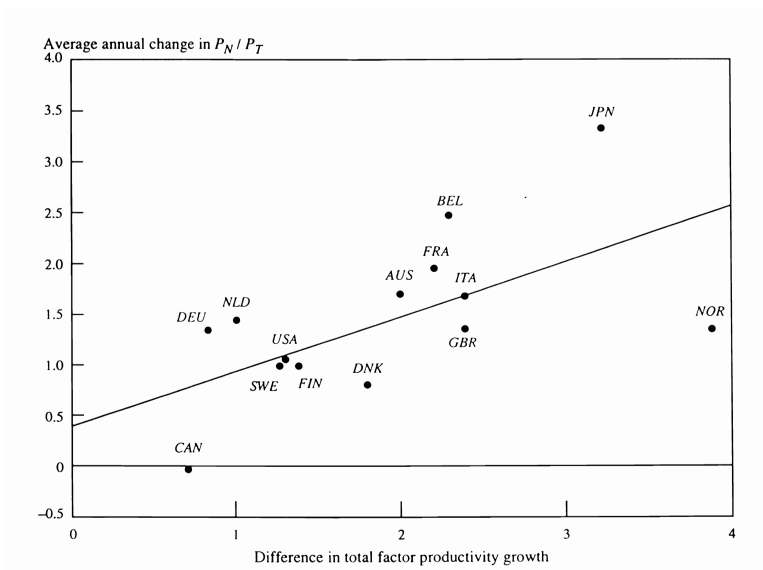
In the foreign country, the relative price of tradables in terms of nontradables is determined in a similar fashion, that is,

$$\frac{P_T^*}{P_N^*} = \frac{a_N^*}{a_T^*}, \tag{9.6}$$

where P_T^*/P_N^* denotes the relative price of tradables in terms of nontradables in the foreign country, and a_T^* and a_N^* denote the labor productivities in the foreign country's traded and nontraded sectors, respectively. To obtain the equilibrium bilateral real exchange rate, $e = E\,P^*/P$, combine equations (9.2), (9.5) and (9.6):

$$e = \frac{\phi(1, a_T^*/a_N^*)}{\phi(1, a_T/a_N)} \tag{9.7}$$

Figure 9.3: Differential Factor Productivity Growth and Changes in the Relative Price of Nontradables



Note: The figure plots the average annual percentage change in the relative price of nontradables in terms of tradables (vertical axis) against the average annual growth in total factor productivity differential between the traded sector and the nontraded sectors (horizontal axis) over the period 1970-1985 for 14 OECD countries. Source: José De Gregorio, Alberto Giovannini, and Holger C. Wolf, "International Evidence on Tradable and Nontradable Inflation," *European Economic Review* 38, June 1994, 1225-1244.

This equation captures the main result of the Balassa-Samuelson model, namely, that deviations from PPP (i.e., variations in e) are due to differences in relative productivity growth rates across countries. In particular, if in the domestic country the relative productivity of the traded sector, a_T/a_N, is growing faster than in the foreign country, then the real exchange rate will appreciate over time (e will fall over time), this is because in the home country nontradables are becoming relatively more expensive to produce than in the foreign country, forcing the relative price of nontradables in the domestic country to grow at a faster rate than in the foreign country.

The relative price of traded goods in terms of nontraded goods, P_T/P_N, can be related to the slope of the production possibility frontier as follows. Let L denote the aggregate labor supply, which we will assume to be fixed. Then the resource constraint in the labor market is

$$L = L_N + L_T$$

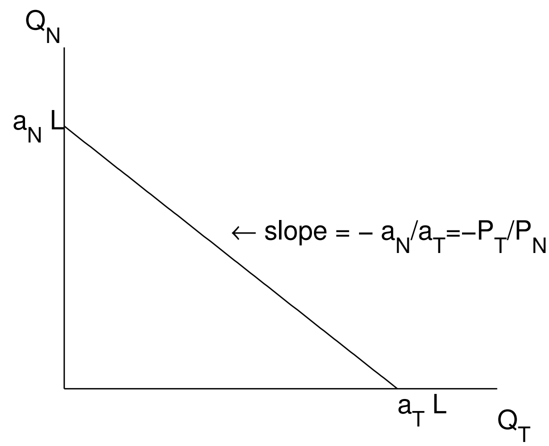
Use equations (9.3) and (9.4) to eliminate L_N and L_T from this expression to get $L = Q_N/a_N + Q_T/a_T$. Now solve for Q_N to obtain the following production possibility frontier (PPF)

$$Q_N = a_N L - \frac{a_N}{a_T} Q_T$$

Figure 9.4 plots the production possibility frontier. The slope of the PPF is

$$\frac{dQ_N}{dQ_T} = -\frac{a_N}{a_T}$$

Figure 9.4: The production possibility frontier (PPF): the case of linear technology



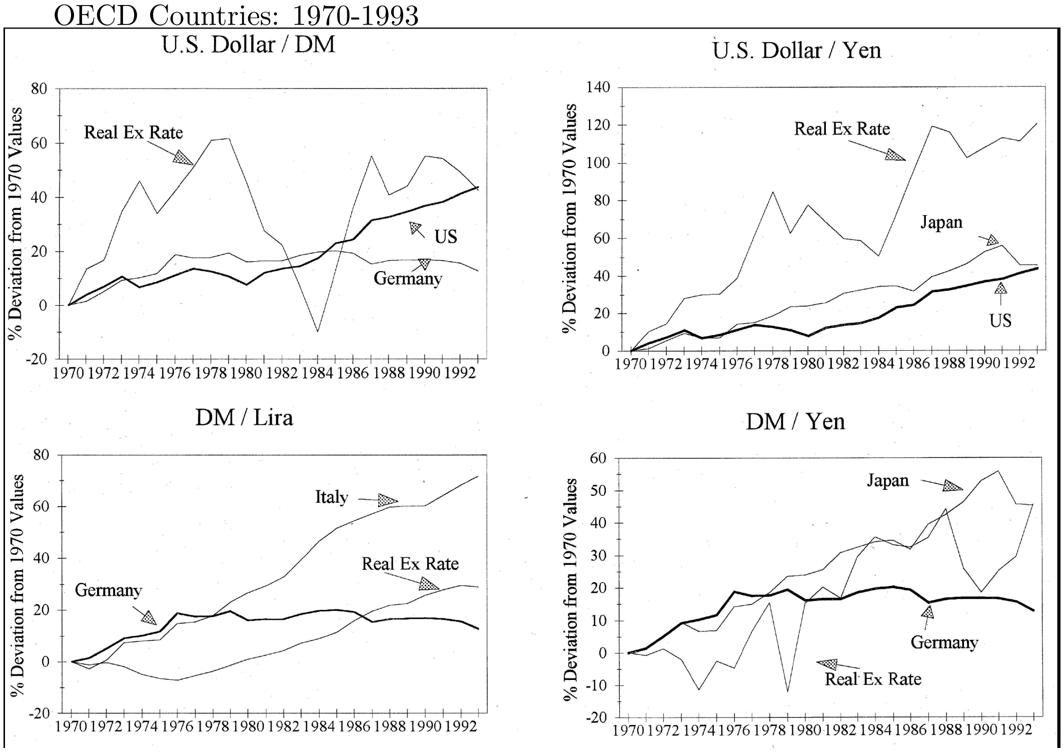
Combining this last expression with equation (9.5), it follows that the slope of the PPF is equal to $-P_T/P_N$.

9.4.1 Application: The Real Exchange Rate and Labor Productivity: 1970-1993

Figure 9.5, reproduced from a quantitative study of productivity and exchange rates by Matthew B. Canzoneri, Robert E. Cumby, and Behzad Diba of Georgetown University,[4] plots bilateral real exchange rates and the ratio of labor productivity in the traded and the nontraded goods sectors for four OECD country pairs. For instance, the top left panel plots $e^{\$/DM} \equiv E^{\$/DM} P^{Germany}/P^{US}$, a_T^{US}/a_N^{US}, and $a_T^{Germany}/a_N^{Germany}$, where DM stands for German mark. As Canzoneri, Cumby, and Diba observe,

[4] Canzoneri, Robert E. Cumby, and Behzad Diba, "Relative Labor Productivity and the Real Exchange Rate in the Long Run: Evidence for a Panel of OECD Countries," *Journal of International Economics* 47, 1999, 245-266.

Figure 9.5: The Real Exchange Rate and Labor Productivity in selected OECD Countries: 1970-1993



Source: Matthew B. Canzoneri, Robert E. Cumby, and Behzad Diba, "Relative Labor Productivity and the Real Exchange Rate in the Long Run: Evidence for a Panel of OECD Countries," *Journal of International Economics* 47, 1999, 245-266.

the figure suggests that the Balassa-Samuelson model has mixed success at explaining real-exchange-rate movements over the period 1970-1993. The Balassa-Samuelson model does a fairly good job at explaining the DM/Lira and the DM/Yen real exchange rates. Between the late 1970s and the early 1990s, both Italy and Japan experienced faster productivity growth in the traded sector relative to the nontraded sector than did Germany. At the same time, as predicted by the Balassa-Samuelson both the Italian lira and the Japanese yen appreciated in real terms vis-à-vis the German mark. On the other hand, in the case of the United States, movements in the real exchange rate seem to be less correlated with changes in relative productivity growth. In the case of the Dollar/DM real exchange rate the observed real appreciation of the dollar in the mid 1980s was not accompanied by a corresponding increase in relative productivity differentials in favor of the U.S. traded sector. In the case of the Dollar/Yen exchange rate, the appreciation in the yen in real terms was, as predicted by the Balassa-Samuelson model, associated with an increase in relative labor productivity in the traded sector in Japan. However, the observed changes in relative labor productivity were too small to explain the extent of the real appreciation of the yen against the dollar.

9.4.2 Application: Deviations from PPP observed between rich and poor countries

Table 9.3 shows the bilateral real exchange rate for a number of countries vis-à-vis the United States. Countries are divided into two groups, poor countries and rich countries. The real exchange rate for a given country,

Table 9.3: The real exchange rate of rich and poor countries, 2005

Country	Real Exchange Rate
Ethiopia	5.4
Bangladesh	5.0
India	4.7
Pakistan	3.4
Unites States	1.0
Germany	0.9
Sweden	0.8
Switzerland	0.6
Japan	0.9

Source: World Economic Outlook Database, IMF, April 2006.

say India, vis-à-vis the United States, $e^{rupee/\$}$ is given by $E^{rupee/\$}P^{US}/P^{I}$, where $E^{rupee/\$}$ is the rupee/dollar nominal exchange rate defined as the price of one dollar in terms of rupee, P^{US} is the price level in the U.S., and P^{I} is the price level in India. The table shows that the real exchange rate in poor countries, $e^{poor/US}$, is typically greater than that in rich countries, $e^{rich/US}$. For example, the Bangladesh/U.S. real exchange rate in 2005 was 5.0, but Switzerland's real exchange rate vis-à-vis the dollar was only 0.6. This means that in 2005 a basket of goods in Switzerland was about 8 (=5.0/0.6) times as expensive as in Bangladesh.

How can we explain this empirical regularity? Note that

$$\frac{e^{poor/US}}{e^{rich/US}} = \frac{\frac{E^{poor/US}P^{US}}{P^{poor}}}{\frac{E^{rich/US}P^{US}}{P^{rich}}} = \frac{E^{poor/US}P^{rich}}{E^{rich/US}P^{poor}} = \frac{E^{poor/rich}P^{rich}}{P^{poor}} = e^{poor/rich}$$

Using equation (9.2), $e^{poor/rich}$ can be expressed as

$$e^{poor/rich} = \frac{\phi(1, P_N^{rich}/P_T^{rich})}{\phi(1, P_N^{poor}/P_T^{poor})}$$

Finally, using the Balassa-Samuelson model, to replace price ratios with relative labor productivities (equation (9.6)), we get

$$e^{poor/rich} = \frac{\phi(1, a_T^{rich}/a_N^{rich})}{\phi(1, a_T^{poor}/a_N^{poor})}$$

Productivity differentials between poor and rich countries are most extreme in the traded good sector, implying that $a_T^{rich}/a_N^{rich} > a_T^{poor}/a_N^{poor}$. So the observed relative productivity differentials can explain why the real exchange rate is relatively high in poor countries.

The Balassa-Samuelson framework is most appropriate to study long-run deviations from PPP because productivity differentials change slowly over time. However, we also observe a great deal of variation in real exchange rates in the short run. The next sections and the following chapter study sources of short-run deviations from PPP.

9.5 Trade Barriers and Real Exchange Rates

In the previous section, deviations from PPP occur due to the presence of nontradables. In this section, we investigate deviations from the law of one price that may arise even when all goods are traded. Specifically, we study deviations from the law of one price that arise because governments impose trade barriers, such as import tariffs, export subsidies, and quotas, that artificially distort relative prices across countries.

Consider, for simplicity, an economy in which all goods are internationally tradable. Suppose further that there are two types of tradable goods, importables and exportables. Importable goods are goods that are either imported or produce domestically but coexist in the domestic market with identical or highly substitutable imported goods. Exportable goods are goods that are produced domestically and sold in foreign and possibly domestic markets. Let the world price of importables be P_M^*, and the world price of exportables be P_X^*. In the absence of trade barriers, PPP must hold for both goods, that is, the domestic prices of exportables and importables must be given by

$$P_X = EP_X^*$$

and

$$P_M = EP_M^*,$$

where E denotes the nominal exchange rate defined as the domestic currency price of one unit of foreign currency. The domestic price level, P, is an average of P_X and P_M. Specifically, assume that P is given by

$$P = \phi(P_X, P_M),$$

where $\phi(\cdot, \cdot)$ is an increasing and homogeneous-of-degree-one function. A similar relation holds in the foreign country

$$P^* = \phi(P_X^*, P_M^*)$$

The bilateral real exchange rate, $e = EP^*/P$, can then be written as

$$e = \frac{E\phi(P_X^*, P_M^*)}{\phi(P_X, P_M)} = \frac{\phi(EP_X^*, EP_M^*)}{\phi(P_X, P_M)} = \frac{\phi(P_X, P_M)}{\phi(P_X, P_M)} = 1,$$

where the second equality uses the fact that ϕ is homogeneous of degree one and the third equality uses the fact that PPP holds for both goods.

Consider now the consequences of imposing a tariff $\tau > 0$ on imports in the home country. The domestic price of the import good therefore increases by a factor of τ, that is,

$$P_M = (1+\tau)EP_M^*.$$

The domestic price of exportables is unaffected by the import tariff. Then the real exchange rate becomes

$$e = \frac{E\phi(P_X^*, P_M^*)}{\phi(P_X, P_M)} = \frac{\phi(EP_X^*, EP_M^*)}{\phi(EP_X^*, (1+\tau)EP_M^*)} < 1,$$

where the inequality follows from the fact that $\phi(\cdot, \cdot)$ is increasing in both arguments and that $1 + \tau > 1$. This expression shows that the imposition of import tariffs leads to an appreciation of the real exchange rate as it makes the domestic consumption basket more expensive. Therefore, one source of deviations from PPP is the existence of trade barriers. One should expect that a trade liberalization that eliminates this type of trade distortions should induce an increase in the relative price of exports over imports goods so e should rise (i.e., the real exchange rate should depreciate).[5]

[5]How would the imposition of an export subsidy affect the real exchange rate?

9.6 Exercises

1. [**PPP Exchange Rates from ICP versus Big Mac Exchange rates**] Analyze to which extend the purchasing power of various currencies against the U.S. dollar implied by Big Mac prices are representative. In particular, compare the purchasing power of a currency against the U.S. dollar obtained in the very careful and detailed study by the International Comparison Program of the World Bank to that implied by the Economist Magazine's Big Mac Index. Specifically, let P_t^* denote the foreign price level, P_t^{US} the U.S. price level, and S_t the nominal exchange rate in dollars per unit of foreign currency.

   (a) Construct a graph where you plot the Big Mac real exchange rate for 2005, defined as $S_t P_t^*/P_t$, on the horizontal axis and the IPC measure of the real exchange rate, which in that publication is referred to as the Price Level Index, on the vertical axis. Data for the Price Level Index for 2005 can be found in the table entitled: 2005 ICP Global Results: Summary Table, starting on page 23 of *G*lobal Purchasing Power Parities and Real Expenditures, 2005 International Comparison Program, The World Bank. Data for the Big Mac Index for 2005 can be downloaded from `http://bigmacindex.org/2005-big-mac-index.html.` When computing the real exchange rate from the Big Mac Index follow the scaling convention used in the IPC table, that is, express your results so that the price level in the United States in 2005 is 100. Your graph should have as many observations as countries that are both in the Big Mac Index table and the ICP table.

(b) Present an insightful discussion of your findings.

(c) Then regress the PLI index onto the Big Mac index, that is, estimate

$$PLI_i = \alpha + \beta BMac_i + \epsilon_i;$$

using ordinary least squares. Report your estimate for β and the R^2 of your regression. Provide a verbal discussion of your findings.

(d) Include a printout of the data that went into the construction of your graph.

2. **Real Exchange Rate Determination**

Consider two countries, say the United States and Japan. Both countries produce tradables and nontradables. Suppose that at some point in time the production technology in the Unites States is described by

$$Q_T^{US} = a_T^{US} L_T^{US}; \text{ with } a_T^{US} = 0.4$$

and

$$Q_N^{US} = a_N^{US} L_N^{US}; \text{ with } a_N^{US} = 0.1,$$

where Q_T^{US} and Q_N^{US} denote, respectively, output of tradables and nontradables in the U.S., a_T^{US} and a_N^{US} denote, respectively, labor productivity in the traded and the nontraded sector, and L_T^{US} and L_N^{US} denote, respectively, the amount of labor employed in the tradable and nontradable sectors in the United States. The total supply of labor in the United States is equal to 1, so that $1 = L_T^{US} + L_N^{US}$. At the same point in time, production possibilities in Japan are given by

$$Q_T^J = 0.2 L_T^J$$

and

$$Q_N^J = 0.2 L_N^J,$$

where the superscript J denotes Japan. The total supply of labor in Japan is also equal to 1. Assume that in each country wages in the traded sector equal wages in the nontraded sector. Suppose that the price index in the United States, which we denote by P^{US}, is given by

$$P^{US} = \sqrt{P_T^{US}}\sqrt{P_N^{US}},$$

where P_T^{US} and P_N^{US} denote, respectively, the dollar prices of tradables and nontradables in the United States. Similarly, the price index in Japan is given by

$$P^J = \sqrt{P_T^J}\sqrt{P_N^J},$$

where Japanese prices are expressed in yen.

(a) Calculate the dollar/yen real exchange rate, defined as $e = S^{\$/¥} P^J / P^{US}$. (The answer to this question is a number.)

(b) Suppose that the U.S. labor productivity in the traded sector, a_T^{US}, grows at a 3 percent rate per year, whereas labor productivity in the nontraded sector, a_N^{US}, grows at 1 percent per year. Assume that labor productivities in Japan are constant over time. Calculate the growth rate of the real exchange rate. Provide an intuitive explanation of your result.

Chapter 10

Changes in Aggregate Spending and the Real Exchange Rate

In the Balassa-Samuelson model studied in section 9.4, the production possibility frontier (PPF) is a *straight line*, which means that the slope of the PPF is the same regardless of the level of production of tradables and nontradables. Because in equilibrium the relative price of tradables in terms of nontradables equals the slope of the PPF, it follows that in the Balassa-Samuelson model the real exchange rate is independent of the level of production of tradables and nontradables. In this section, we will study a more realistic version of the model, known as the TNT model, in which the PPF is a concave function. As a result of this modification, the slope of the PPF, and therefore also the relative price P_T/P_N, depend on the composition of output (i.e., on the amount of tradable goods and nontradable goods produced in the economy). In turn, the composition of output is determined in part by the desired level of aggregate spending.

The TNT model has three building blocks: The *production possibility frontier*, which describes the production side of the economy; the *income expansion path*, which summarizes the aggregate demand for goods; and *international borrowing and lending*, which allows agents to shift consumption across time. In the Balassa-Samuelson model neither the second nor the third building blocks are needed for the determination of the real exchange rate because in that model the PPF alone determines the real exchange rate.

10.1 The production possibility frontier

Consider an economy that produces traded and nontraded goods with labor as the only factor input. Specifically, the production functions are given by

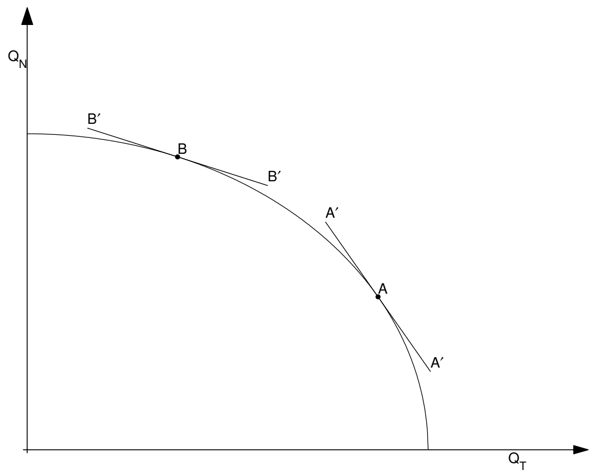
$$Q_T = F_T(L_T) \tag{10.1}$$

and

$$Q_N = F_N(L_N), \tag{10.2}$$

where Q_T and Q_N denote output of traded and nontraded goods, respectively, and L_T and L_N denote labor input in the traded and nontraded sectors. The production functions $F_T(\cdot)$ and $F_N(\cdot)$ are assumed to be increasing and concave, that is, $F_T' > 0$, $F_N' > 0$, $F_T'' < 0$, and $F_N'' < 0$. The assumption that the production functions are concave means that the

Figure 10.1: The production possibility frontier (PPF): the case of decreasing marginal productivity of labor



marginal productivity of labor is decreasing in the amount of labor input used.[1]

The total supply of labor in the economy is assumed to be equal to L, which is a positive constant. Therefore, the allocation of labor across sectors must satisfy the following resource constraint:

$$L_T + L_N = L. \tag{10.3}$$

The two production functions along with this resource constraint can be combined into a single equation relating Q_N to Q_T. This relation is the production possibility frontier of the economy and is shown in figure 10.1. Consider the following example:

[1] Compare these production functions to those of the Balassa-Samuelson model. In the Balassa-Samuelson model, the production functions are $F_T(L_T) = a_T L_T$ and $F_N(L_N) = a_N L_N$. Thus, in that model $F_T' = a_T > 0$ and $F_N' = a_N > 0$, which means that the marginal product of labor is constant in both sectors, or, equivalently, that $F_T'' = F_N'' = 0$.

$$Q_T = \sqrt{L_T}$$

and

$$Q_N = L_N.$$

Solve both production functions for labor to obtain $L_T = Q_T^2$ and $L_N = Q_N$. Use these two expressions to eliminate L_T and L_N from the resource constraint $L_T + L_N = L$ to obtain the PPF

$$Q_N = L - Q_T^2.$$

This expression describes a negative and concave relationship between Q_T and Q_N like the one depicted in figure 10.1.

More generally, the fact that the production functions given in (10.1) and (10.2) display decreasing marginal productivity of labor implies that the PPF is concave toward the origin. The slope of the PPF, dQ_N/dQ_T, indicates the number of units of nontraded output that must be given up to produce an additional unit of traded output. That is, the slope of the PPF represents the cost of producing an additional unit of tradables in terms of nontradables. As Q_T increases, the PPF becomes steeper, which means that as Q_T increases, it is necessary to sacrifice more units of nontraded output to increase traded output by one unit. The slope of the PPF is given by the ratio of the marginal products of labor in the two sectors, that is,

$$\frac{dQ_N}{dQ_T} = -\frac{F_N'(L_N)}{F_T'(L_T)} \tag{10.4}$$

This expression makes it clear that the reason why the PPF becomes steeper as Q_T increases is that as Q_T increases so does L_T and thus the marginal productivity of labor in the traded sector, $F_T'(L_T)$ becomes smaller, while the marginal productivity of labor in the nontraded sector, $F_N'(L_N)$, increases as Q_N and L_N decline.

The slope of the PPF can be derived as follows. Differentiate the resource constraint (10.3) to get

$$dL_T + dL_N = 0$$

or

$$\frac{dL_N}{dL_T} = -1$$

This expression says that, because the total amount of labor is fixed, any increase in labor input in the traded sector must be offset by a one-for-one reduction of labor input in the nontraded sector. Now differentiate the production functions (10.1) and (10.2)

$$\begin{aligned} dQ_T &= F_T'(L_T)dL_T \\ dQ_N &= F_N'(L_N)dL_N \end{aligned}$$

Taking the ratio of these two equations and using the fact that $dL_N/dL_T = -1$ yields equation (10.4).

The slope of the PPF indicates how many units of nontradables it costs to produce one additional unit of tradables. In turn, the relative price of tradables in terms of nontradables, P_T/P_N, measures the relative revenue of selling one unit of traded good in terms of nontraded goods. Profit-maximizing firms will choose a production mix such that the relative revenue of selling an additional unit of tradables in terms of nontradables equals the relative cost of tradables in terms of nontradables. That is, firms will produce at a point at which the slope of the PPF equals (minus) the relative price of tradables in terms of nontradables:

$$\frac{F_N'(L_N)}{F_T'(L_T)} = \frac{P_T}{P_N} \tag{10.5}$$

Suppose that the real exchange rate, P_T/P_N is given by minus the slope of the line $A'A'$, which is $-P_T^o/P_N^o$ in figure 10.1. Then firms will choose to produce at point A, where the slope of the PPF is equal to the slope of $A'A'$. Consider now the effect of a real exchange rate appreciation, that is, a decline in P_T/P_N.[2] The new relative price is represented by the slope of the line $B'B'$, which is flatter than $A'A'$. In response to the decline in the relative price of tradables in terms of nontradables, firms choose to produce less tradables and more nontradables. Specifically, the new production mix is given by point B, located northwest of point A.

The optimality condition (10.5) can be derived more formally as follows. Consider the problem faced by a firm in the traded sector. Its profits are given by revenues from sales of tradables, $P_TF_T(L_T)$, minus the cost of production, wL_T, where w denotes the wage rate, that is,

$$\text{profits in the traded sector} = P_TF_T(L_T) - wL_T$$

[2]Note that here we use the term ”real exchange rate” to refer to the relative price of tradables in terms of nontradables, P_T/P_N.

The firm will choose an amount of labor input that maximizes its profits. That is, it will choose L_T such that

$$P_T F_T'(L_T) - w = 0.$$

This first-order condition is obtained by taking the derivative of profits with respect to L_T and setting it equal to zero. The first-order condition says that the firm will equate the value of the marginal product of labor to the marginal cost of labor, w. A similar relation arises from the profit-maximizing behavior of firms in the nontraded sector:

$$P_N F_N'(L_N) - w = 0$$

Combining the above two first-order conditions to eliminate w yields equation (10.5).

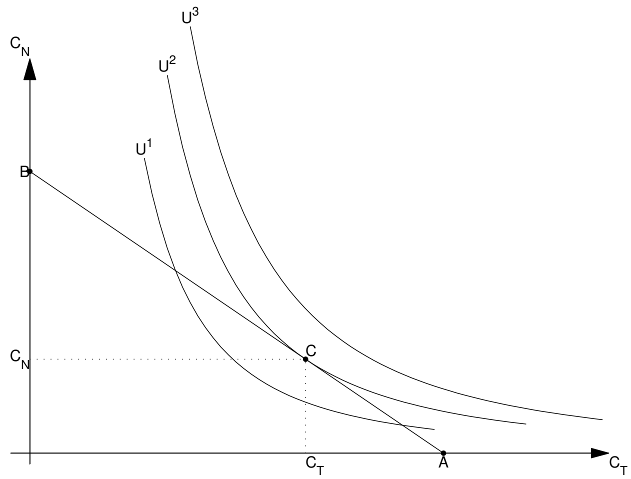
10.2 The income expansion path

Consider now the household's demand for tradable and nontradable consumption. In each period, households derive utility from consumption of traded and nontraded goods. In particular, their preferences are described by the following single-period utility function

$$U(C_T, C_N) \tag{10.6}$$

where $U(\cdot, \cdot)$ is increasing in both arguments. Figure 10.2 shows the in-

Figure 10.2: The household's problem in the TNT model



difference curves implied by the utility function given in equation (10.6). The indifference curves are as usual downward sloping and convex toward the origin reflecting the fact that households like both goods and that the marginal rate of substitution of tradables for nontradables (the slope of the indifference curves) is decreasing in C_T. Also, because more is preferred to less, the level of utility increases as one moves northeast in the space (C_T, C_N). Thus, for example, in figure 10.2 the level of utility is higher on the indifference curve U^3 than on the indifference curve U^1.

Suppose the household has decided to spend the amount Y on consumption. How will the household allocate Y to purchases of each of the two goods? The household's budget constraint is given by

$$P_T C_T + P_N C_N = Y. \tag{10.7}$$

This constraint says that total expenditures on traded and nontraded consumption purchases must equal the amount the household chose to spend on consumption this period, Y. In figure 10.2 the budget constraint is given by the straight line connecting points A and B. If the household chooses to consume no nontraded goods, then it can consume Y/P_T units of traded goods (point A in the figure). On the other hand, if the household chooses to consume no traded goods, it can consume Y/P_N units of nontraded goods (point B in the figure). The slope of the budget constraint is given by $-P_T/P_N$.

The household chooses C_T and C_N so as to maximize its utility function (10.6) subject to its budget constraint (10.7). The maximum attainable level of utility is reached by consuming a basket of goods on an indifference curve that is tangent to the budget constraint, point C in the figure. At point C, the slope of the indifference curve equals the slope of the budget constraint. To derive this result algebraically, solve (10.7) for C_N and use the resulting expression, $C_N = Y/P_N - P_T/P_N C_T$, to eliminate C_N from (10.6). Then the household's problem reduces to choosing C_T so as to maximize

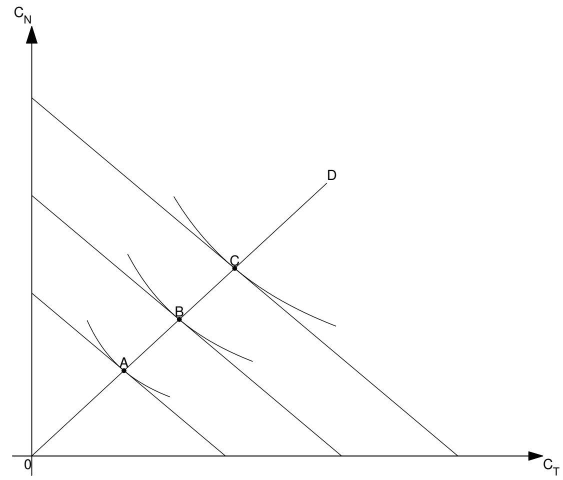
$$U\left(C_T, \frac{Y}{P_N} - \frac{P_T}{P_N} C_T\right)$$

The first-order condition of this problem is obtained by taking the derivative with respect to C_T and equating it to zero:

$$U_T\left(C_T, \frac{Y}{P_N} - \frac{P_T}{P_N} C_T\right) - \frac{P_T}{P_N} U_N\left(C_T, \frac{Y}{P_N} - \frac{P_T}{P_N} C_T\right) = 0$$

where $U_T(\cdot, \cdot)$ and $U_N(\cdot, \cdot)$ denote the partial derivatives of the utility func-

Figure 10.3: The income expansion path



tion with respect to its first and second argument, respectively (or the marginal utilities of consumption of tradables and nontradables). Rearranging terms and using the fact that $Y/P_N - P_T/P_N C_T = C_N$ yields:

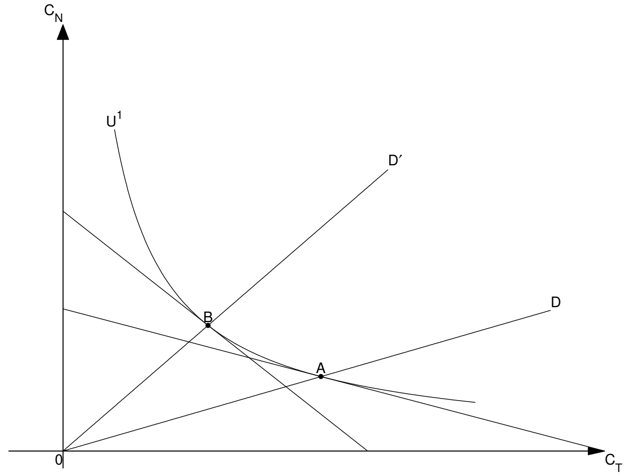
$$\frac{U_T(C_T, C_N)}{U_N(C_T, C_N)} = \frac{P_T}{P_N} \tag{10.8}$$

The left hand side of this expressions is (minus) the slope of the indifference curve (also known as the marginal rate of substitution between traded and nontraded goods). The right hand side is (minus) the slope of the budget constraint.

Consider the household's optimal consumption choice for different levels of income. Figure 10.3 shows the household's budget constraint for three different levels of income, Y_1, Y_2, and Y_3, where $Y_1 < Y_2 < Y_3$. As income increases, the budget constraint shifts to the right in a parallel fashion. It shifts to the right because given for any given level of consumption of one of the goods, an increase in income allows the household to consume more of the other good. The shift is parallel because the relative price between tradables and nontradables is assumed to be unchanged (recall that the slope of the budget constraint is $-P_T/P_N$). We will assume that both goods are normal, that is, that in response to an increase in income, households choose to increase consumption of both goods. This assumption implies that the optimal consumption basket associated with the income level Y_2 (point B in the figure) contains more units of both tradable and nontradable goods than the consumption bundle associated with the lower income Y_1 (point A in the figure), that is, point B is located northeast of point A. Similarly, consumption of both traded and nontraded goods is higher when income is equal to Y_3 (point C in the figure) than when income is equal to Y_2. The *income expansion path* (IEP) is the locus of optimal consumption baskets corresponding to different levels of income, holding constant the relative price of traded and nontraded goods. Clearly, points A, B, and C must lie on the same income expansion path given by the line $\overline{\mathrm{OD}}$ in figure 10.3.

Income expansion paths have four important characteristics: First, if both goods are normal, then income expansion paths are upward sloping. Second, income expansion paths must begin at the origin. This is because if income is nil, then consumption of both goods must be zero. Third, at the point of intersection with a given IEP, all indifference curves have the same slope. This is because each IEP is constructed for a given relative price P_T/P_N, and because at the optimal consumption allocation, the slope of the indifference curve must be equal to the relative price of the two goods. Fourth, an increase in the relative price of traded in terms of nontraded

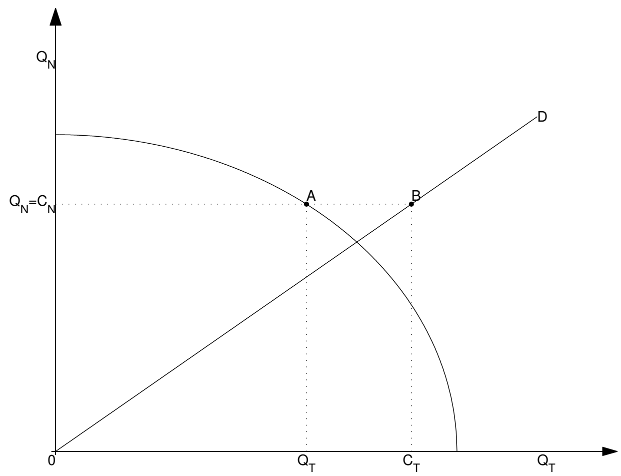
Figure 10.4: The income expansion path and a depreciation of the real exchange rate



goods, P_T/P_N, produces a counterclockwise rotation of the IEP.

The intuition behind this last characteristic is that if the relative price of tradables in terms of nontradables goes up, households consume relatively less tradables and more nontradables. Figure 10.4 shows two income expansion paths, $\overline{\mathrm{OD}}$ and $\overline{\mathrm{OD}'}$. The relative price underlying $\overline{\mathrm{OD}}$ is lower than the relative price underlying $\overline{\mathrm{OD}'}$. To see this, consider the slope of any indifference curve as it intersects each of the two IEPs. Take for example the indifference curve U^1 in figure 10.4. At the point of intersection with $\overline{\mathrm{OD}}$ (point A in the figure), U^1 is flatter than at the point of intersection with $\overline{\mathrm{OD}'}$ (point B). Because at point A the slope of U^1 is equal to the relative price underlying $\overline{\mathrm{OD}}$, and at point B the slope of U^1 is equal to the relative price underlying $\overline{\mathrm{OD}'}$, it follows that the relative price associated with $\overline{\mathrm{OD}'}$ is higher than the relative price associated with $\overline{\mathrm{OD}}$.

Figure 10.5: Partial Equilibrium

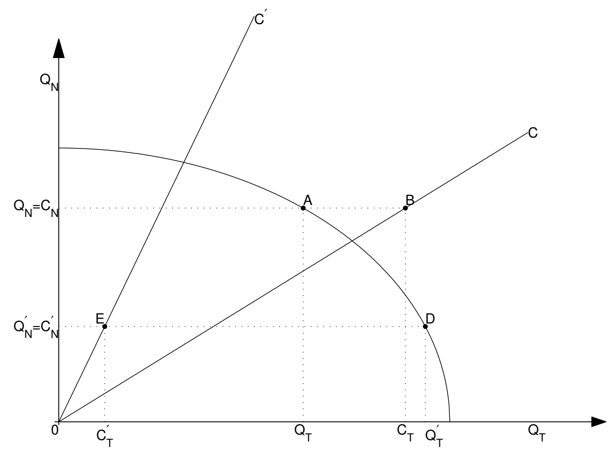


10.3 Partial equilibrium

We can now put together the first two building blocks of the model, the production possibility frontier and the income expansion path, to analyze the determination of production, consumption and the real exchange rate given the trade balance. Figure 10.5 illustrates a partial equilibrium. Suppose that in equilibrium production takes place at point A on the PPF. The equilibrium real exchange rate, P_T/P_N, is given by the slope of the PPF at point A. Suppose that the IEP corresponding to the equilibrium real exchange rate is the line $\overline{\text{OD}}$. By definition, nontraded goods cannot be imported or exported. Therefore, market clearing in the nontraded sector requires that production equals consumption, that is,

$$C_N = Q_N \tag{10.9}$$

Figure 10.6: Partial equilibrium: a real exchange rate depreciation



Given consumption of nontradables, the IEP determines uniquely the level of consumption of tradables (point B in the figure). Because our model does not feature investment in physical capital or government purchases, the trade balance is simply given by the difference between production and consumption of tradables,

$$TB = Q_T - C_T \tag{10.10}$$

In the figure, the trade balance is given by the horizontal distance between points A and B. Because in the figure consumption of tradables exceeds production, the country is running a trade balance deficit.

Consider now the effect of a depreciation of the real exchange rate, that is, an increase in P_T/P_N. Figure 10.6 illustrates this situation. The economy is initially producing at point A and consuming at point B. Because in equilibrium the slope of the PPF must equal the real exchange rate, the depreciation of the real exchange rate induces a change in the production mix to a point like D, where the PPF is steeper than at point A. This shift in the composition of production has a clear intuition: as the price of tradables goes up relative to that of nontradables, firms find it profitable to expand production of traded goods at the expense of nontraded goods. On the demand side of the economy, the real exchange rate depreciation causes a counterclockwise rotation in the income expansion path from $\overline{\text{OC}}$ to $\overline{\text{OC'}}$. Having determined the new production position and the new IEP, we can easily determine the new equilibrium consumption basket (point E in the figure) and trade balance (the horizontal distance between points D and E).

Summing up, in response to the real exchange rate depreciation, the economy produces more tradables and less nontradables, and consumes less tradables as well as nontradables. As a result of the expansion in the production of tradables and the contraction in consumption of tradables, the economy ends up generating a smaller trade balance deficit. In fact, in the case shown in figure 10.6 the trade balance becomes positive. Figure 10.7 depicts the relationship between trade deficits, the real exchange rate, consumption, and production.

The TNT model can help understand the effects of external shocks that force countries to sharply adjust their current accounts. An example of this type of shock is the Debt Crisis of Developing Countries of the early 1980s, which we will discuss in more detail in chapter 11. In 1982, adverse conditions in international financial markets caused credit to dry up for highly indebted countries, particularly in Latin America. As a consequence, debtor countries, which until that moment were running large current ac-

Figure 10.7: Partial equilibrium: endogenous variables as functions of the trade deficit

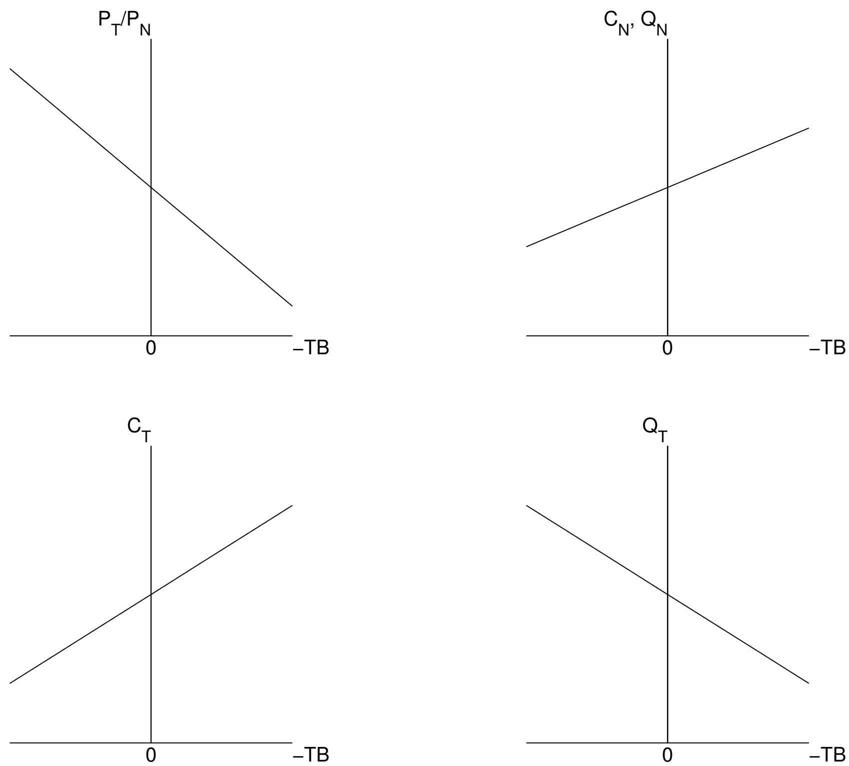


Table 10.1: Chile, trade balance and real exchange rate depreciation, 1979-1985

Year	Δe %	$\frac{TB}{GDP}$ %
1979		-1.7
1980		-2.8
1981		-8.2
1982	20.6	0.3
1983	27.5	5.0
1984	5.1	1.9
1985	32.6	5.3

count deficits, were all of the sudden forced to generate large trade balance surpluses in order to be able to service their debts. As predicted by the TNT model, the required external adjustment produced sharp real exchange rate depreciations, large contractions in aggregate spending, and costly reallocations of production away from the nontraded sector and toward the traded sector. Table 10.1 illustrates the effect of the Debt Crisis on Chile's trade balance and real exchange rate. In terms of the TNT model, the intuition behind the effect of the Debt Crisis on the affected developing countries is clear. In response to the shutdown of external credit, countries needed to generate trade balance surpluses to pay interest and principal on existing foreign debt. In order to generate a trade balance surplus, aggregate spending must decline. Given the relative price of tradables in terms of nontradables, P_T/P_N, households will cut consumption of both traded and nontraded goods. At the same time, given the relative price of tradables in terms of nontradables, production of nontradables should be unchanged. This means that an excess supply of nontradables would emerge. The only way that the market for nontradables can clear is if the relative price of nontradables falls—that is, if the real exchange rate depreciates—inducing firms to produce less nontradables and households to consume more nontradables.

The tools developed thus far allow us to determine all variables of interest given the trade deficit, but do not tell us how the trade deficit itself is determined. Another way of putting this is that our model has more variables than equations. The equilibrium conditions of our model are: equations (10.1), (10.2), and (10.3) describing the PPF, equation (10.5), which ensures that the real exchange rate equals the slope of the PPF, equation (10.8) describing the IEP, equation (10.9), which guarantees market clearing in the nontraded sector, and equation (10.10), which defines the trade balance. These are 7 equations in 8 unknowns: Q_N, Q_T, L_N, L_T, C_N, C_T, TB, and P_T/P_N. To "close" the model, we need a theory to determine TB. More specifically, we need a theory that explains households' consumption decisions over time. In the next section, we merge the *static* partial equilibrium model developed in this section with the *intertemporal* approach to the current account studied in earlier chapters to obtain a *dynamic general equilibrium* model.

10.4 General equilibrium

To determine the equilibrium level of the trade balance, we introduce an intertemporal dimension to the TNT model. Assume that households live for two periods and have preferences described by the following intertemporal utility function

$$U(C_{T1}, C_{N1}) + \beta U(C_{T2}, C_{N2}),$$

where C_{T1} and C_{N1} denote, respectively, consumption of tradables and non-tradables in period 1, and C_{T2} and C_{N2} denote the corresponding variables in period 2. The function $U(\cdot,\cdot)$ is the single period utility function given in (10.6), and $0 < \beta < 1$ is a constant parameter, called subjective discount factor, which determines the value households assign to future utility.

In the previous section, we deduced that, all other things constant, in equilibrium both C_T and C_N are increasing functions of the trade deficit, $-TB$ (see figure 10.7). Thus, we can define an *indirect* utility function $\tilde{U}(-TB) \equiv U(C_T, C_N)$ with C_T and C_N replaced by increasing functions of $-TB$. Clearly, the indirect utility function is increasing in $-TB$, because both C_T and C_N are increasing in $-TB$. We can therefore write the intertemporal utility function as

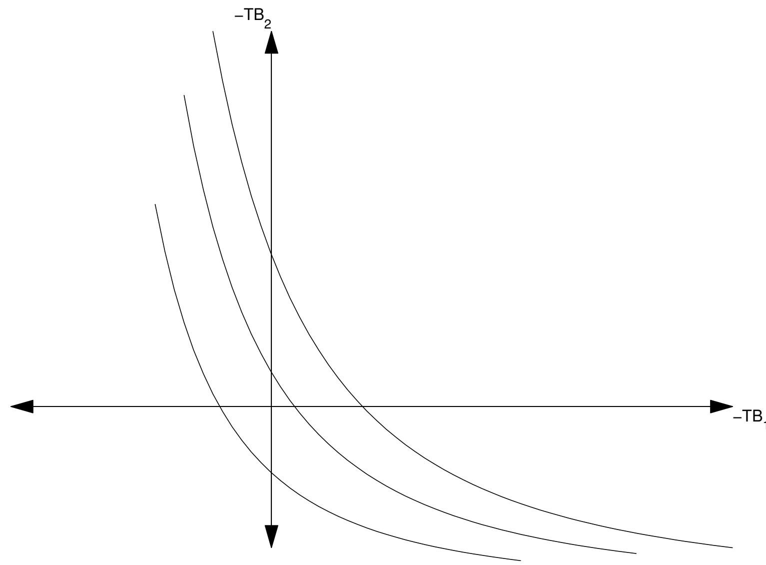
$$\tilde{U}(-TB_1) + \beta\tilde{U}(-TB_2) \tag{10.11}$$

Figure 10.8 shows the indifference curves associated with the indirect utility function (10.11). The indifference curves have the conventional form. They are downward sloping and convex to the origin. As one moves northeast in the space $(-TB_1, -TB_2)$ utility increases.

The household's budget constraint in period 1 is given by

$$C_{T1} + \frac{P_{N1}}{P_{T1}} C_{N1} + B_1^* = (1 + r_0)B_0^* + Q_{T1} + \frac{P_{N1}}{P_{T1}} Q_{N1}$$

Figure 10.8: The indirect utility function: indifference curves



The right hand side of this expression represents the sources of wealth of the household measured in terms of tradables. The households initial asset holdings including including interest are $(1 + r_0)B_0^*$, where B_0^* are initial holdings of foreign bonds denominated in units of traded goods, and r_0 is the return on the initial holdings of foreign bonds. The second source of wealth is the value of output in period 1, $Q_{T1} + (P_{N1}/P_{T1})Q_{N1}$, measured in terms of tradables. Note that we are measuring nontraded output in terms of tradables by multiplying it by the relative price of nontradables in terms of tradables. The left hand side of the budget constraint represents the uses of wealth. The household allocates its wealth to purchases of consumption goods, $C_{T1} + \frac{P_{N1}}{P_{T1}}C_{N1}$, and to purchases of foreign bonds, B_1^*. In equilibrium the market clearing condition in the nontraded sector requires that consumption of nontradables be equal to production of nontradables, that is,

$C_{N1} = Q_{N1}$ (equation (10.9)). In addition, we have that $TB_1 = Q_{T1} - C_{T1}$ (equation (10.10)). Thus, the household's budget constraint in period 1 can be written as

$$-TB_1 + B_1^* = (1 + r_0)B_0^*$$

Similarly, in period 2 the budget constraint takes the form

$$-TB_2 + B_2^* = (1 + r_1)B_1^*,$$

where r_1 denotes the domestic interest rate paid on holdings of the foreign bond between periods 1 and 2. Foreign bonds are measured in terms of tradables. Thus, r_1 is the real interest rate in terms of tradables. [3] We will assume that the economy is small and that there is free capital mobility, so that the domestic interest rate on tradables must be equal to the world interest rate, r^*, that is,

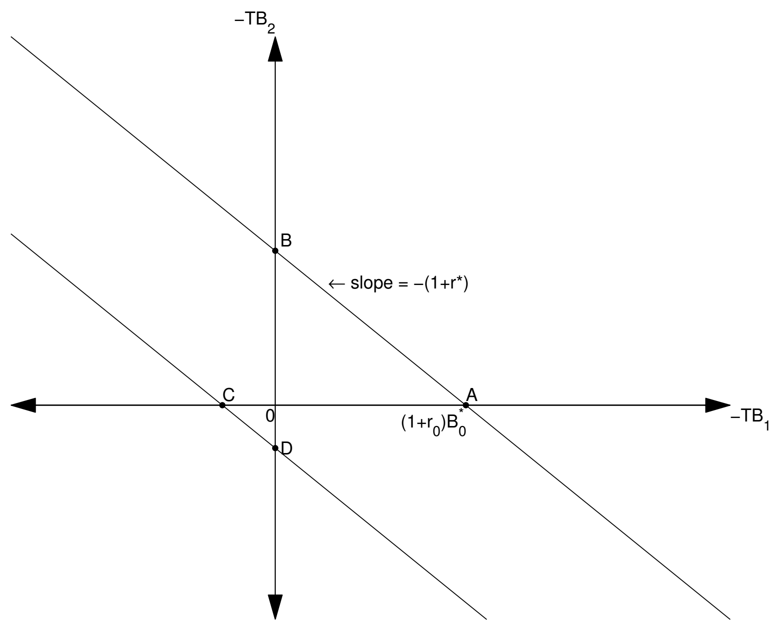
$$r_1 = r^*.$$

By the no-Ponzi-game constraint $B_2^* \geq 0$ and the fact that no household is willing leave outstanding assets in period 2, we have

$$B_2^* = 0$$

[3]The interest rate in terms of tradables indicates how many units of tradables one receives next periods for each unit of tradables invested today. On the other hand, the interest rate in terms of nontradables represents the amount of nontradables one receives tomorrow per unit of nontradables invested today, and is given by $(1+r_1)(P_{N1}/P_{T1})/(P_{N2}/P_{T2})$. To see why this is so, note that 1 unit of nonntradables in period 1 buys P_{N1}/P_{T1} units of tradables in period 1, which can be invested at the rate r_1 to get $(1 + r_1)P_{N1}/P_{T1}$ units of tradables in period 2. In turn each unit of tradables in period 2 can be exchanged for P_{T2}/P_{N2} units of nontradables in that period.

Figure 10.9: The intertemporal budget constraint



Combining the above four equations to eliminate B_1^*, B_2^*, and r_1, we get the following lifetime budget constraint

$$-TB_1 - \frac{TB_2}{1+r^*} = (1+r_0)B_0^* \tag{10.12}$$

This budget constraint says that the present discounted value of current and future trade deficits must be equal to the household's initial foreign asset holdings including interest payments. This way of writing the lifetime budget constraint should be familiar from earlier lectures. Indeed, we derived an identical expression in the context of a single-good, endowment economy (equation (3.7)). Figure 10.9 shows the lifetime budget constraint (10.12). The slope of the budget constraint is negative and given by $-(1+r^*)$. If

$-TB_2 = 0$, then in period 1 the economy can run a trade deficit equal to its entire initial wealth, that is, $-TB_1 = (1+r_0)B_0^*$ (point A in the figure). Alternatively, if $-TB_1 = 0$, then $-TB_2 = (1+r^*)(1+r_0)B_0^*$ (point B). The fact that at point A the trade deficit in period 1, $-TB_1$, is positive means initial asset holdings are positive ($(1+r_0)B_0^* > 0$). But this need not be the case. If the country was an initial debtor ($(1+r_0)B_0^* < 0$), then the budget constraint would be a line like the one connecting points C and D. In this case, point C is on the negative range of the horizontal axis indicating that even if the trade balance is zero in period 2, the country must generate a trade surplus in period 1 in order to pay back its initial debt.

In equilibrium, households choose trade deficits in periods 1 and 2 so as to maximize their lifetime utility. This situation is attained at a point on the budget constraint that is tangent to an indifference curve (point A in figure 10.10). This implies that at the equilibrium allocation, the slope of the indifference curve is equal to the slope of the budget constraint. To derive this result formally, solve the budget constraint (10.12) for $-TB_1$ and use the result to eliminate $-TB_1$ from the indirect utility function (10.11), which yields

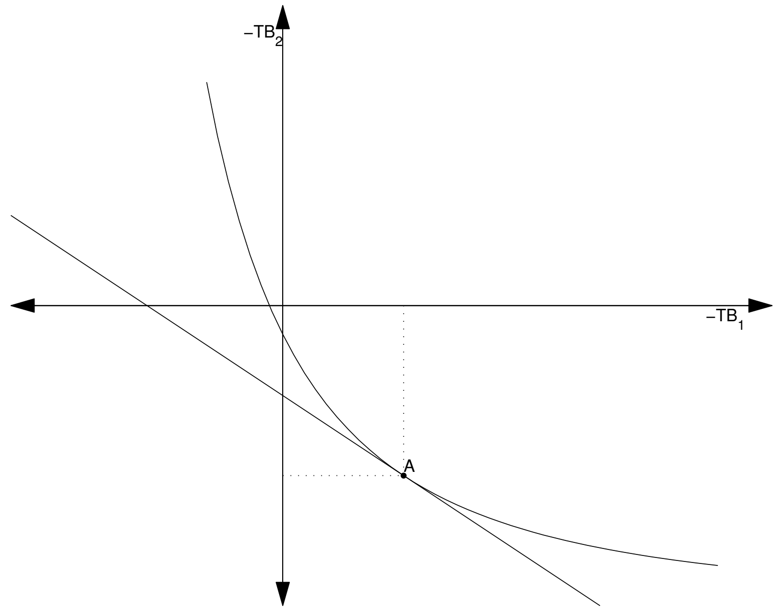
$$\tilde{U}\left((1+r_0)B_0^* - \frac{-TB_2}{1+r^*}\right) + \beta\tilde{U}(-TB_2).$$

To find the optimal level of the trade deficit in period 2, take the derivative of this expression with respect to $-TB_2$ and set it equal to zero, to get

$$\tilde{U}'\left((1+r_0)B_0^* - \frac{-TB_2}{1+r^*}\right)\left(\frac{-1}{1+r^*}\right) + \beta\tilde{U}'(-TB_2) = 0$$

Rearranging terms and taking into account that $(1+r_0)B_0^* - (-TB_2)/(1+$

Figure 10.10: General equilibrium



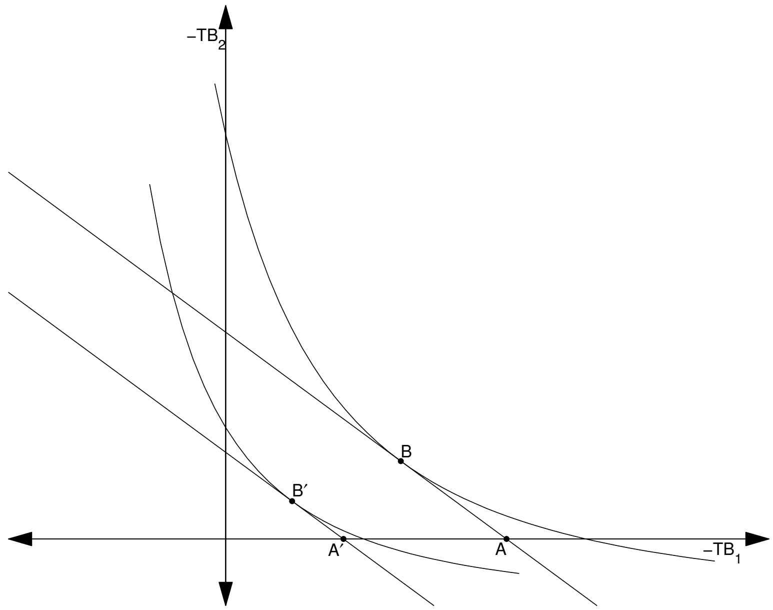
$r^*) = -TB_1$ we obtain

$$\frac{\tilde{U}'(-TB_1)}{\beta\tilde{U}'(-TB_2)} = 1 + r^*. \tag{10.13}$$

The left hand side of this equation is (minus) the slope of the indifference curve, and the right hand side is (minus) the slope of the budget constraint.

With this optimality condition we have "closed" the model. By closing the model we mean that we now have as many equilibrium conditions as we have endogenous variables. To recapitulate, in the previous subsection we obtained 7 equilibrium conditions for each period (equations (10.1), (10.2), (10.3), (10.5), (10.8), (10.9), and (10.10)) and 8 unknowns for each period (Q_N, Q_T, L_N, L_T, C_N, C_T, TB, and P_T/P_N). In this subsection, we obtained 2 additional equilibrium conditions, equations (10.12) and (10.13),

Figure 10.11: A negative wealth shock



by studying the intertemporal choice problem of the household.[4] Therefore, we now have 16 equations in 16 unknowns, so that the model is closed. In the next subsection we put the model to work by using it to address a number of real life questions.

10.5 Wealth shocks and the real exchange rate

Consider the effect of a decline in a country's net foreign asset position on the real exchange rate and the trade balance. Figure 10.11 depicts the situation of a country that has a positive initial net foreign asset position ($(1 + r_0)B_0^*$) given by point A. The equilibrium is given by point B where the intertemporal budget constraint is tangent to an indifference curve. A

[4]Note that equations (10.12) and (10.13) do not introduce any additional unknowns.

decline in the initial net foreign asset position causes a parallel shift in the budget constraint to the left. In the figure, the change in the initial wealth position is given by the distance between points A and A'. The new equilibrium is given by point B', where the trade deficits in both periods are lower. The intuition behind this result is straightforward: as the country becomes poorer it must reduce aggregate spending. Households choose to adjust in both periods because in that way they achieve a smoother path of consumption over time.

Having established the effect of the wealth shock on the trade balance, we can use figure 10.7 to deduce the response of the remaining endogenous variables of the model. The negative wealth effect produces a decline in consumption of tradables and nontradables in both periods. This result makes sense, given that the economy has become poorer. In addition, the real exchange rate depreciates, or tradables become more expensive relative to nontradables. This change in relative prices is necessary in order to induce firms to produce less nontradables when the demand for this type of good falls. Finally, output increases in the traded sector and declines in the nontraded sector. Thus, the improvement in the trade balance is the result of both a decline in consumption and an expansion in production of tradables.

Wealth shocks provide an example of long-lasting deviations from PPP that arise even if productivity is not changing, and thus represent an alternative explanation of movements in the real exchange rate to the one offered by the Balassa-Samuelson model.

Are the predictions of the TNT model consistent with the observed response of countries that faced large wealth shocks? An example of a large negative wealth shock is World War II. For example, in Great Britain large military spending and structural damage wiped out much of the country's net foreign asset position and resulted in a protracted depreciation of the pound vis-à-vis the U.S. dollar.

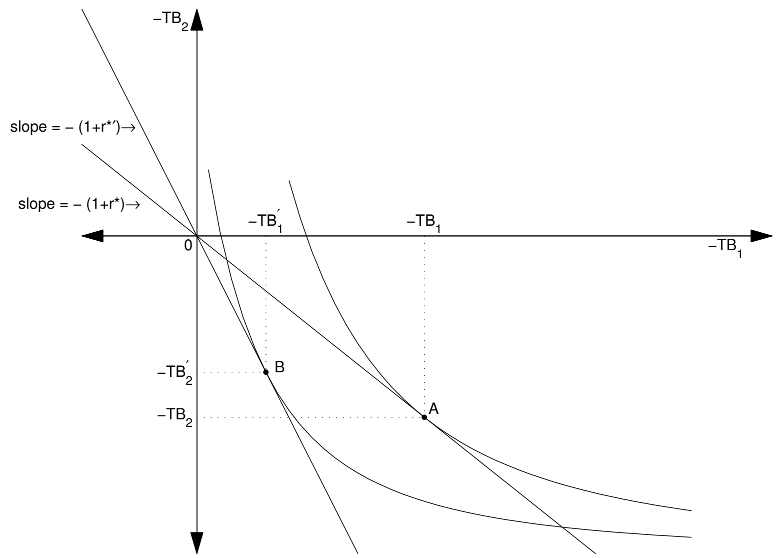
10.6 World interest rate shocks

It has been argued that in developing countries, variations in the real exchange rate are to a large extent due to movements in the world interest rate. For example, Guillermo Calvo, Leonardo Leiderman, and Carmen Reinhart studied the commovement between real exchange rates and U.S. interest rates for ten Latin American countries between 1988 and 1992.[5] They find that around half of the variance in real exchange rates can be explained by variations in U.S. interest rates. In particular, they find that in periods in which the world interest rate is relatively low, the developing countries included in their study experience real exchange rate appreciations. Conversely, periods of high world interest rates are associated with depreciations of the real exchange rate.

Is the TNT model consistent with the observed negative correlation between interest rates and the real exchange rate? Consider a small open economy, which, for simplicity, is assumed to start with zero initial wealth. Suppose further that the country is borrowing in period 1. The situation is

[5]G. Calvo, L. Leiderman, and C. Reinhart, "Capital Inflows and Real Exchange Rate Appreciation in Latin America: The Role of External Factors," *International Monetary Fund Staff Papers,* Vol. 40, March 1993, 108-151.

Figure 10.12: An increase in the world interest rate



illustrated in figure 10.12. The budget constraint crosses the origin, reflecting the fact that the initial net foreign asset position is nil. In the initial situation, the world interest rate is r^*. The equilibrium allocation is given by point A. The country is running a trade balance deficit in period 1 and a surplus in period 2. Suppose now that the world interest rate increases from r^* to $r^{*\prime} > r^*$. The higher interest rate causes a clockwise rotation of the budget constraint. The new equilibrium is point B, where the steeper budget constraint is tangent to an indifference curve. At point B, the economy is running a smaller trade deficit in period 1 than at point A. The improvement in the trade balance is the consequence of two reinforcing effects. First, the increase in the interest rate produces a substitution effect that induces households to postpone consumption and increase savings. Second, because the economy is borrowing in period 1, the increase in the interest rate makes domestic households poorer, thus causing a decline in aggregate spending.

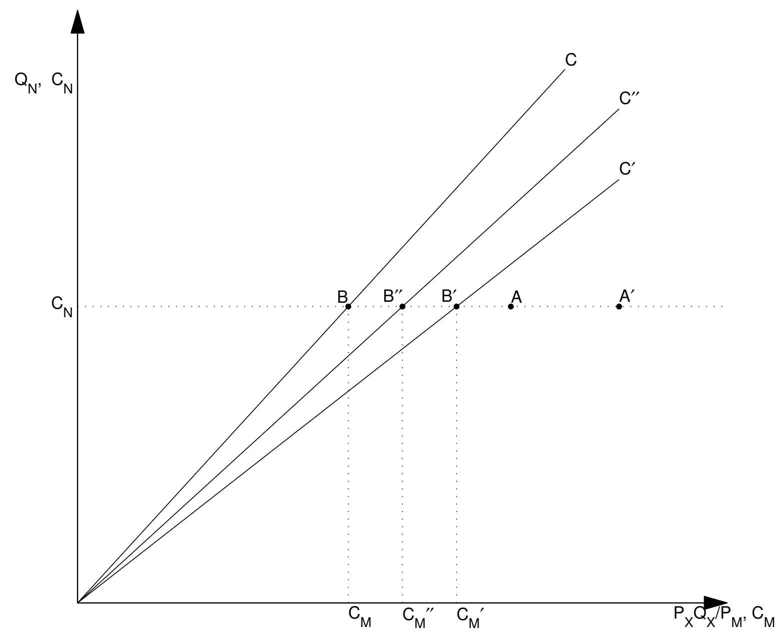
It follows from figure 10.7 that the decline in the trade balance in period 1 caused by the interest rate hike is accompanied by a decline in consumption of tradables and nontradables, an expansion in traded output and a contraction in the nontraded sector. Finally, the real exchange rate depreciates. The TNT model is therefore consistent with the observation that high interest rates are associated with real depreciations of the exchange rate.

10.7 Terms-of-trade shocks

In order to incorporate terms-of-trade (TOT), we must augment the model to allow for two kinds of traded goods: importables and exportables. We will assume, as we did in our earlier discussion of terms of trade (subsection 3.3), that the country's supply of tradables is exported and not consumed, and that all traded goods consumed by domestic households are imported. The distinction between importables and exportables makes matters more complicated. To compensate, we will simplify the model's structure by assuming that the supplies of tradables and nontradables are exogenous. That is, we will study the effects of TOT shocks in an endowment economy. The only difference with our earlier treatment of TOT shocks is therefore the presence of nontradable goods.

Households consume importable goods and nontraded goods, and are endowed with fixed quantities of exportables and nontradables. Let C_M denote consumption of importables and Q_X the endowment of exportable goods. Let P_X/P_M denote the terms of trade, defined as the relative price

Figure 10.13: An improvement in the terms of trade



of exportables in terms of importables. In this endowment economy, the PPF collapses to a single point, namely, the endowment of tradables and nontradables (Q_X, Q_N). Point A in figure 10.13 represents the value of the economy's endowment. In order to measure imports and exports in the same units on the horizontal axis, the endowment of exportables is expressed in terms of importables by multiplying Q_X by the terms of trade, P_X/P_M. Suppose that in equilibrium the economy is running a trade surplus equal to the horizontal distance between points A and B. It follows that the income expansion path, given by the locus $\overline{\mathrm{OC}}$, must cross point B. The real exchange rate, now defined as P_M/P_N, can be read of the slope of the indifference curve at point B.

Suppose that the economy experiences a permanent improvement in the terms of trade, that is, an increase in P_X/P_M in both periods. Because the value of the endowment of exportables went up, point A in figure 10.13 shifts horizontally to the right to point A'. At the same time, the permanent TOT shock is likely to have a negligible effect on the trade balance. The reason is that a permanent increase in the TOT is equivalent to a permanent positive income shock, to which households respond by increasing consumption in both periods in the same magnitude as the increase in income, thus leaving the trade balance unchanged. The fact that the trade balance is unchanged implies that in the new equilibrium consumption of importables must increase in the same magnitude as the increase in the value of the endowment of tradables. The new consumption point is given by B' in the figure. The distance between A and B is the same as the distance between A' and B'. The new income expansion path must go through point B'. This means that the IEP rotates clockwise, or, equivalently, that the real exchange rate appreciates (P_M/P_N goes down) in response to the improvement in TOT. The intuition behind this result is clear. The permanent increase in income caused by the improvement in TOT induces households to demand more of both goods, importables and nontradables. Because the supply of nontradables is fixed, the relative price of nontradables (the reciprocal of the real exchange rate) must increase to discourage consumption of nontradables, thereby restoring equilibrium in the nontraded sector.

Suppose now that the improvement in the terms of trade is temporary rather than permanent, that is, that P_X/P_M increases only in period 1. In this case, households will try to smooth consumption by saving part of the positive income shock in period 1. As a result the trade balance in period 1 improves. In terms of figure 10.13, the new consumption position, point B", is such that the distance between B" and B is smaller than the distance between A′ and A, reflecting the improvement in the trade balance. Therefore, as in the case of a permanent TOT shock, in response to a temporary TOT shock the IEP shifts clockwise. However, the rotation is smaller than under a permanent TOT shock. Consequently, the real exchange rate appreciation is also smaller under a temporary TOT shock than under a permanent one.

10.8 Exercises

1. An Economy With Tradable and Non-Tradable Goods

   Consider a two-period, small, open economy. In period 1, households receive an endowment of 6 units of tradable goods and 9 units of non-traded goods. In period 2, households receive 13.2 units of tradables and 9 units of nontradables ($Q_1^T = 6$, $Q_2^T = 13.2$, and $Q_1^N = Q_2^N = 9$). Households start period 1 with no assets or liabilities ($B_0^* = 0$). The country enjoys free access to world financial markets, where the prevailing interest rate is 10 percent ($r^* = 0.1$). Suppose that the household's preferences are defined over consumpiton of tradable and nontradable goods in periods 1 and 2, and are described by the following utility function,

   $$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

   where C_i^T and C_i^N denote, respectively, consumption of tradables and nontradables in period $i = 1, 2$. Let p_1 and p_2 denote the relative prices of nontradables in terms of tradables in periods 1 and 2, respectively.

   (a) Write down the budget constraints of the household in periods 1 and 2.

   (b) Derive the household's intertemporal budget constraint. Assign this expression the number (1).

   (c) The household chooses consumption of tradables and nontradables in periods 1 and 2 to maximize its lifetime utility function subject to its intertemporal budget constraint. Find the optimality conditions associated with this problem. To this end, begin by solving (1) for C_1^T and use the resulting expression to eliminate C_1^T from the lifetime utility function. Derive the resulting lifetime utility with respect to C_1^N, C_2^T, and C_2^N, and assign the resulting expressions the numbers (2), (3), and (4), respectively. Express (2), (3), and (4) in terms of (a subset) of C_1^T, C_1^N, C_2^T, and C_2^N.

(d) Write down the market clearing conditions in the nontraded goods market in periods 1 and 2.

(e) Use these market clearing conditions to eliminate C_1^N, C_2^N, Q_1^N, Q_2^N, p_1, and p_2 from the intertemporal budget constraint (1). Assign the number (5) to the resulting intertemporal resource constraint.

(f) Now solve (3) and (5) for C_1^T and C_2^T as functions of only Q_1^T, Q_2^T, and r^*.

(g) Calculate the net foreign asset position of the economy at the end of period 1, B_1^*.

(h) Compute for period 1 the equilibrium levels of the current account balance and the relative price of nontradables in terms of tradables.

(i) Calculate for period 2 the equilibrium levels of the current account balance and the relative price of nontradables in terms of tradables. Explain intuitively why the relative price of nontradables changes over time.

(j) Assume that the domestic consumer price index in period $t = 1, 2$, denoted P_t, is defined by $P_t = \sqrt{P_t^T P_t^N}$, where P_t^T and P_t^N denote the nominal prices of tradables and nontradables in period $t = 1, 2$, respectively. Similarly, suppose that the foreign consumer price index is given by $P_t^* = \sqrt{P_t^{T*} P_t^{N*}}$, where the superscript $*$ denotes foreign variables. Foreign nominal prices are expressed in terms of foreign currency. Assume that PPP holds for tradable goods. Finally, suppose that the foreign relative price of nontradables in terms of tradables equals unity in both periods. Compute the real exchange rate in periods 1 and 2.

(k) **An External Crisis:** Let us sketch a scenario like the one that took place during the Argentine debt crisis of 2001 by assuming that because of fears that the country will not repay its debts in period 2, foreign lenders refuse to extend loans to the domestic economy in period 1. Answer the questions in items 7 through 10 under these new (adverse) circumstances. Compute the equilibrium interest rate. Provide an intuitive explanation of your results.

(l) Compute real GDP in period 1 under the crisis and no-crisis scenarios. Consider two alternative measures of real GDP: GDP measured in terms of tradable goods and GDP measured in terms of the basket of goods whose price is the consumer price index P_1. What measure is more economically sensible? Why?

(m) **Crisis Relief Policy:** Suppose the Inter American Development Bank (IADB) decided to implement a transfer (gift) to Argentina to ameliorate the effects of the external crisis. Specifically, suppose that the IADB gives Argentina a transfer of F units of tradable goods in period 1. Use the utility function given above to compute the size of F that would make Argentineans as happy as in the no-crisis scenario. Express F as a percentage of the country's crisis/no-aid GDP in period 1.

2. Sudden Stops With Downward Wage Rigidity

Consider a two-period, small, open economy. Households are endowed with 10 units of tradables in period 1 and 13 units in period 2 ($Q_1^T = 10$ and $Q_2^T = 13$). The country interest rate is 10 percent, or $r = 0.1$, the nominal exchange rate is fixed and equal to 1 in both periods ($E_1 = E_2 = 1$), and the nominal wage equals 8.25 in both periods ($W_1 = W_2 = 8.25$). Nominal wages are downwardly rigid. Suppose the economy starts period 1 with no assets or debts carried over from the past ($B_0^* = 0$). Suppose that the household's preferences are defined over consumption of tradable and nontradable goods in periods 1 and 2, and are described by the following utility function,

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where C_i^T and C_i^N denote, respectively, consumption of tradables and nontradables in period $i = 1, 2$. Let p_1 and p_2 denote the relative prices of nontradables in terms of tradables in periods 1 and 2, respectively. Households supply inelastically $\bar{h} = 1$ units of labor to the market each period. Finally, firms produce nontradable goods using labor as the sole input. The production technology is given by $Q_1^N = h_1^\alpha$ and $Q_2^N = h_2^\alpha$ in periods 1 and 2, respectively, where Q_i^N and h_i denote, respectively, nontradable output and hours employed in period $i = 1, 2$. The parameter α is equal to 0.75.

(a) Compute the equilibrium levels of consumption of tradables and the trade balance in periods 1 and 2.

(b) Compute the equilibrium levels of employment and nontradable output in period 1.

(c) Suppose now that the country interest rate increases to 32 percent. Calculate the equilibrium levels of consumption of tradables, the trade balance, consumption of nontradables, and the level of unemployment, all for period 1. Provide intuition.

(d) Given the situation in the previous question, calculate the minimum devaluation rate consistent with full employment. Explain.

Chapter 11

The Macroeconomics of External Debt

11.1 The debt crisis of developing countries of the 1980s

In 1982, the government of Mexico announced that it could no longer meet its external financial obligations. This episode marked the beginning of what today is known as the Developing Country Debt Crisis. Mexico's decision was followed by similar measures by other highly indebted developing countries, particularly in Latin America. In this section we present an analytical overview of the events leading to the Debt Crisis, its economic consequences, and its reversal with the capital inflows of the 1990s.

The fact that many countries were affected simultaneously suggests that international factors played an important role in the financial crisis of the early 1980s.

A number of external factors led to a large accumulation of debt by developing countries in the second half of the 1970s. The sharp oil price increase in 1973-74 led to huge deposits by middle eastern countries in international banks. Flushed with funds, commercial banks were eager to lend. In addition, in general, bankers in industrialized countries strongly felt that developing countries could never go bankrupt. Two other external factors were important in explaining the unusual amount of capital that flowed to Latin America and other developing countries in the late 1970s: low real interest rates and large growth in exports.

There were also domestic government policies in Latin America that encouraged borrowing in the late 1970s. First, financial liberalization, led to large expansions in lending, as interest rate controls in the banking sector were removed. In some countries, such as Argentina and Chile, the government provided loan guarantees. Thus, domestic banks had incentives to borrow at very high rates and invested in risky projects. In fact, it was as if the government was subsidizing foreign borrowing by domestic banks.

A second domestic factor was the exchange rate policy followed by a number of Latin American countries. In the mid 1970s, countries in the Southern Cone of Latin America pegged their currencies to the U.S. dollar as a way to fight inflation. This policy resulted in a significant real exchange rate appreciation (i.e., in a fall in $S \cdot P^*/P$) and large current account deficits. Households expanded purchases of imported goods, especially durables such as cars and electrodomestics.

In the early 1980s, there was a dramatic change in the economic environ-

Table 11.1: Interest rates in the late 1970s and early 1980s

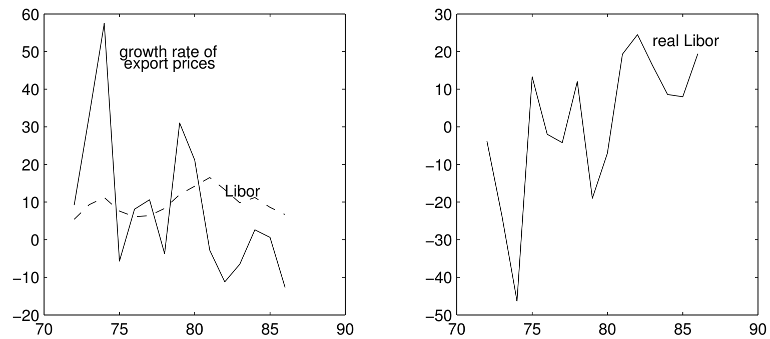
Year	Nominal LIBOR
1978	8.3
1979	12.0
1980	14.2
1981	16.5

Source: Andres Bianchi et al., "Adjustment in Latin America, 1981-86," in V. Corbo, M. Goldstein, and M. Khan, ed., *Growth Oriented Adjustment Programs,* Washington, D.C.: International Monetary Fund and The World Bank, 1987.

ment. World interest rates increased sharply due to the anti-inflationary policy in the U.S. led by Federal Reserve chairman Paul Volker (see table 11.1). In addition, the terms of trade deteriorated for the debtor countries as raw material prices fell. As a result, the real interest rate faced by developing countries rose dramatically (see figure 11.1).

Debtor countries were highly vulnerable to the rise in world interest rates because much of the debt carried a floating rate. In Latin America, 65% of the foreign debt had a floating rate. Thus, debt service increased rapidly and unexpectedly in the early 1980s. The combination of higher interest rates and lower export prices resulted in sharp increases in interest payments relative to export earnings in highly indebted developing countries (see table 11.2). External lending to developing countries and inflows of foreign investment abruptly stopped in 1982. For all developing countries, new lending was 38 billion in 1981, 20 billion in 1982, and only 3 billion in

Figure 11.1: Interest rates and export prices in Latin America (1972-1986)



Note: The real Libor rate is constructed by subtracting the rate of change in export prices from the nominal Libor rate.

Source: Andres Bianchi et al., "Adjustment in Latin America, 1981-86," in V. Corbo, M. Goldstein, and M. Khan, ed., *Growth Oriented Adjustment Programs,* Washington, D.C.: International Monetary Fund and The World Bank, 1987.

Table 11.2: Interest payments in selected Latin American countries. Average 1980-81.

Country	Percent of Debt at floating rate	Interest Payment to Exports ratio (%)
Argentina	58	15
Brazil	64	28
Colombia	39	16
Chile	58	28
Mexico	73	19
All Latin America	65	28

Source: Andres Bianchi et al., "Adjustment in Latin America, 1981-86," in V. Corbo, M. Goldstein, and M. Khan, ed., *Growth Oriented Adjustment Programs,* Washington, D.C.: International Monetary Fund and The World Bank, 1987.

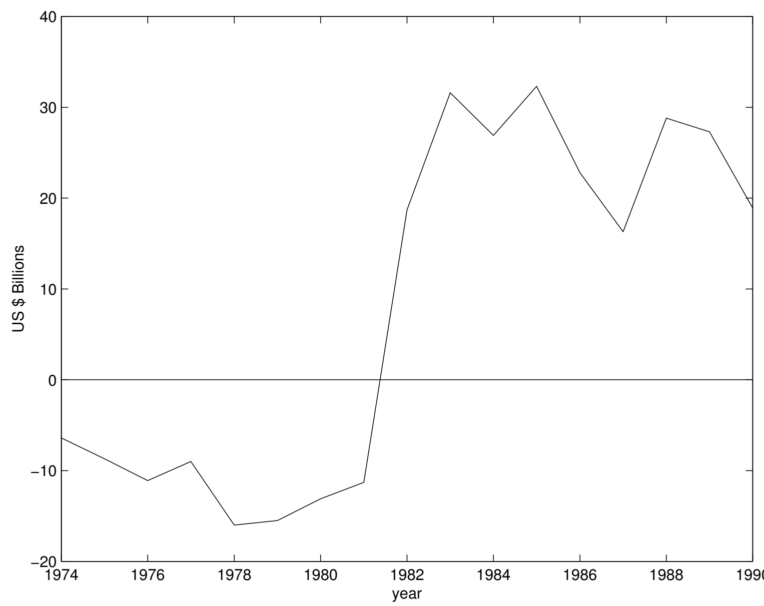
1983.

Domestic factors also contributed to the slowdown in capital inflows. The exchange rate policy of pegging the domestic currency to the U.S. dollar followed by countries in the Southern Cone of Latin America was believed to be unsustainable, in part because governments did fail to implement the required fiscal reforms. As a result, by the early 1980s expectations of real depreciation of the domestic currency induced domestic residents to invest in foreign assets (capital flight). In addition, the risky projects taken up by banks following the financial liberalization of the late 1970s and encouraged by government guarantees resulted in systemic banking failures.

As a result of the shutdown of foreign credit, countries were forced to generate large current account surpluses in order to continue to service, at least in part, their external obligations (see figure 11.2).

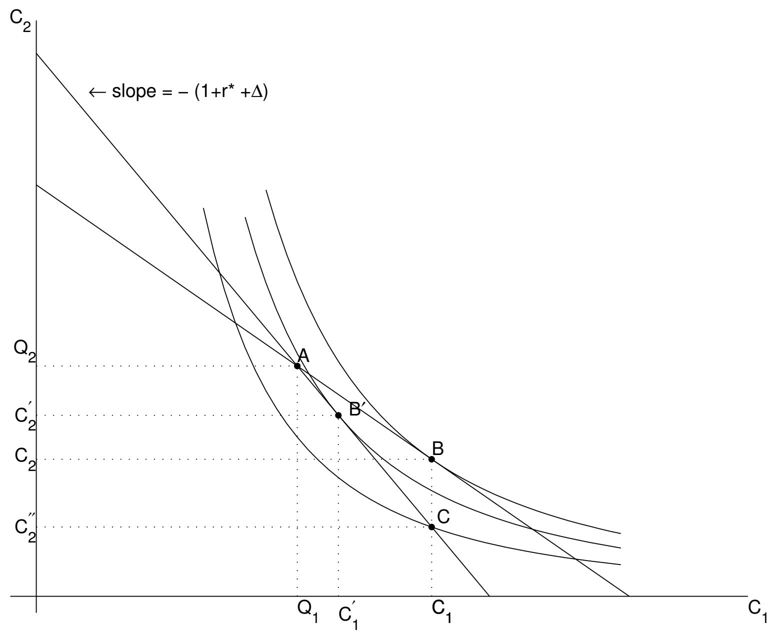
What does our model say about the macroeconomic consequences of a sharp world interest rate increase for a debtor country whose debt is at floating rates? Figure 11.3 depicts an endowment economy that starts with a zero initial net foreign asset position ($(1+r_0)B_0^* = 0$). The endowment point, (Q_1,Q_2), is given by point A in the figure. The initial equilibrium is at point B, where the economy is running a current account deficit (or borrowing from abroad an amount) equal to $Q_1 - C_1$ in period 1. The situation in period 1 resembles the behavior of most Latin American countries in the late 1970s, which, taking advantage of soft international credit conditions borrowed heavily in international capital markets. Consider now an increase in the world interest rate like the one that took place in the early 1980s. The interest rate hike entailed an increase in the amount of resources needed

Figure 11.2: The trade balance in Latin America (1974-1990)



Source: Economic Commission for Latin America and the Caribbean (ECLAC), Preliminary Overview of the Economy of Latin America and the Caribbean, Santiago, Chile, December 1990.

Figure 11.3: Floating Interest Rates and Current Account Adjustment



to service not only newly assumed obligations but also *existing* debts. This is because, as we argued above, most of the developing country debt was stipulated at *floating* rates. In terms of our graph, the increase in the interest rate from r^* to $r^* + \Delta$ causes a clockwise rotation of the budget constraint around point A.

We assume that households took on their debt obligations under the expectations that the world interest rate would be r^*. We also assume that the interest rate hike takes place *after* the country assumes its financial obligations in period 1. However, in period 2 the country must pay the higher interest rate on the financial obligations assumed in period 1 because those obligations stipulated a floating rate. Therefore, households cannot reoptimize and choose point B', featuring a lower trade deficit—and hence lower foreign debt—in period 1. They are stuck with $TB_1 = Q_1 - C_1$. This means that the new position of the economy is point C on the new budget constraint and vertically aligned with point B. The increase in the world interest rate forces the country to generate a large trade balance in period 2, given by $Q_2 - C_2''$ in order to service the debt contracted in period 1. Note that the trade surplus in period 2 is much larger than it would have been had the country been able to re-optimize its borrowing in period 1 $(Q_2 - C_2')$. It is clear from figure 10.7 that the improvement in the trade balance leads to a depreciation of the real exchange rate and a contraction in aggregate spending. The response of the economy in period 2 captures pretty well the adjustment that took place in most Latin American countries in the wake of the Debt Crisis. Figure 11.2 documents the spectacular trade balance reversal that took place in Latin America in 1982. Table 10.1, shows that in Chile, the improvement in the current account in the aftermath of the debt crisis was accompanied by a dramatic (and traumatic) real exchange rate depreciation. The Chilean experience is not atypical. Large real depreciations were observed across Latin America after 1982.

11.2 The resurgence of capital inflows to developing countries in the 1990s

In the 1990s, developing countries in Asia and Latin America experienced a resurgence of capital inflows. About \$670 billion of foreign capital flowed to these countries in the 5 years from 1990 to 1994, as measured by the total balance on the financial account. This is 5 times larger than the \$133 billion of total inflows during the previous 5 years.

An article by Guillermo Calvo, Leonardo Leiderman, and Carmen Reinhart analyzes the causes of the resurgence of capital inflows to developing countries in the 1990s and argues that a number of factors were at work.[1] The widespread nature of the phenomenon suggests that global factors were especially important. Many of these factors are the same that led to high capital inflows to the region in the late 1970s. Domestic factors also played a role in determining the magnitude and composition of capital flows.

First, interest rates in international financial markets in the 1990s were relatively low. After peaking in 1989, interest rates in the U.S. declined steadily in the early 1990s. In 1992 interest rates reached their lowest level since the 1960s. This attracted capital to high-yield investments in Asia and Latin America. Second, in the early 1990s, the U.S., Japan, and several countries in Western Europe were in recession, which implied that they offered fewer investment opportunities. Third, rapid growth in international diversification and international capital market integration, facilitated in part by financial deregulation in the U.S. and Europe, allowed mutual funds and life insurance companies to diversify their portfolios to include emerging market assets. Fourth, many developing countries made progress toward improving relations with external creditors. Fifth, many developing countries adopted sound fiscal and monetary policies and market-oriented reforms such as trade and capital liberalization (Chile, Bolivia, and Mexico in the 1980s, Argentina, Brazil, Ecuador, and Peru in the 1990s). Finally, there

[1] See G. Calvo, L. Leiderman, and C. Reinhart, "Inflows of Capital to Developing Countries in the 1990s," *Journal of Economic Perspectives,* 10, Spring 1996, 123-139.

Table 11.3: Selected recipients of large capital inflows: macroeconomic performance 1988-1994

Country	Year Capital Inflow began	Cumulative RER appreciation	Average CA/GDP
Asia			
Indonesia	1990	-6.2	-2.5
Malaysia	1989	-3.9	-4.8
Philippines	1992	20.9	-4.2
Thailand	1988	1.9	-6.0
Latin America			
Argentina	1991	20.1	-3.1
Brazil	1992	57.9	-.2
Chile	1990	13.5	-1.8
Colombia	1991	37.1	-4.2
Mexico	1989	23.4	-6.8

Source: "Inflows of Capital to Developing Countries in the 1990s" by G. Calvo, L. Leiderman, and C. Reinhart, *Journal of Economic Perspectives*, Spring 1996.

seemed to be what some researchers call contagion. The opening of a large developing economy to capital markets (like Mexico in the late 1980s) can produce positive externalities that facilitate capital inflows to other neighboring countries.

As shown in table 11.3, the capital inflows of the 1990s produced a number of important macroeconomic consequences, which are strikingly similar to those that paved the way for the debt crisis in the late 1970s: (1) The counterpart of the surge in capital inflows was a large increase in current account deficits, which materialized via investment booms and declines in savings. (2) In Latin America, the surge in capital inflows led to large real exchange appreciations. By contrast, in Asia such appreciation was

Table 11.4: The evolution of the debt/GNP ratio in selected countries, 1980-1985

	$\frac{D}{GDP}$		
	1980	1982	1985
Argentina	.48	.84	.84
Brazil	.31	.36	.49
Mexico	.30	.53	.55

Source: Jeffrey D. Sachs and Felipe Larrain B., *Macroeconomics in the Global Economy,* Prentice Hall, Englewood Cliffs, New Jersey, 1993, Table 22-9.

observed only in the Philippines. (3) The decline in savings was associated with increases in consumption of (mostly imported) durable goods. (4) A significant fraction of capital inflows were channeled to accumulation of foreign exchange reserves by central banks.

11.3 The Debt Burden

A country's debt burden can be measured by its debt-to-GDP ratio,

$$\text{Debt burden} = \frac{D}{GDP},$$

where D denotes the country's stock of external debt and GDP denotes gross domestic product, both measured in terms of tradables. A notable characteristic of the debt crisis was that the debt burden of developing countries rose rather than fell. Table 11.4 shows that the debt burden of Argentina, Brazil, and Mexico was 18 to 36 percentage points higher in 1985 than in

1980. The reason why the observed increase in the debt-to-GDP ratio is surprising is that, as we discussed in the previous section, with the onset of the debt crisis the flow of capital to developing countries came to an abrupt halt. Therefore, the observed rise in the debt burden must have been driven by a decline in GDP rather than an increase in debt.

The reason for the sharp decline in GDP is, among other factors, that large real exchange rate depreciations lead to a decline in the value of domestic output in terms of tradables. Domestic output in terms of tradables is the sum of tradable output and nontradable output measured in terms of tradables, that is,

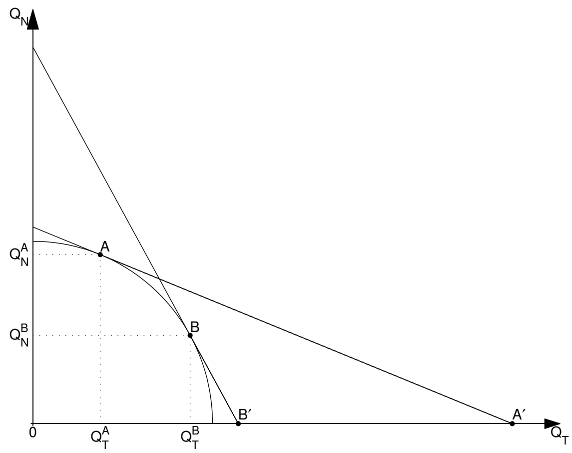
$$\text{GDP in terms of tradables} = Q_T + \frac{P_N}{P_T} Q_N.$$

In response to a real exchange rate depreciation the production of tradables increases and that of of nontradables declines. The value of domestic output of nontradables measured in terms of tradables falls because both Q_N and P_N/P_T fall. On the other hand, production of tradables increases.

How can we determine that the net effect on output in terms of tradables is negative? Let's use the TNT model developed in chapter 10. Consider a small open economy that experiences a sharp deterioration of its real exchange rate. Suppose that initially the country produces at point A in figure 11.4. The equilibrium real exchange rate is given by the negative of the slope of the PPF at point A and GDP in terms of tradables is given by point A′, which is the sum of Q_T^A and $(P_N^A/P_T^A)Q_N^A$.[2] Suppose now that the

[2]To see that point A′ represents GDP in terms of tradables, note that the line connecting A and A′ has slope $-P_T^A/P_N^A$ and crosses the point (Q_T^A, Q_N^A); thus such line can

Figure 11.4: The effect of a real depreciation on the value of GDP in terms of tradables



real exchange rate depreciates and as a consequence equilibrium production takes place at point B on the PPF. The new real exchange rate P_T^B/P_N^B is equal to the negative of the slope of the PPF at point B. As the relative price of tradables rises, production of tradables increases from Q_T^A to Q_T^B and that of nontradables falls from Q_N^A to Q_N^B. The new value of GDP in terms of tradables is given by point B′, which is equal to $Q_T^B + (P_N^B/P_T^B)Q_N^B$. A real exchange rate depreciation thus causes a decline in the value of a country's GDP in terms of tradables and as a consequence implies that the country must spend a larger fraction of its GDP in servicing the external debt.

be written as the pairs (x, y) satisfying $y = Q_N^A - \frac{P_T^A}{P_N^A}(x - Q_T^A)$. We are looking for the intersection of this line with the x axis, that is, for the value of x corresponding to $y = 0$. Setting $y = 0$ we get $x = Q_T^A + (P_N^A/P_T^A)Q_N^A$.

11.4 Debt Reduction Schemes

Soon after the debt crisis of 1982, it became clear to debtor countries, creditors, and multinational organizations, such as the IMF and the World Bank, that full repayment of the developing country debt was no longer realistic and policy makers started to think about debt reduction schemes as a possible solution to the debt crisis.[3]

By the late 1980s the debt of many developing countries was trading in the secondary market at significant discounts, often as low as 50 percent of its face or par value, reflecting the fact that market participants thought that the likelihood that the country would ever be able to fully repay its debt was very low. At the time many policy makers and economists argued that in such a situation it would be best to "face reality" and reduce a country's debt to what it would be able to pay. The idea was that the face value of the outstanding debt should be adjusted so that the debt would be trading around par and the adjustment should take the form of creditors forgiving part of the debt. This idea was not very often implemented because typically it is not in the creditor's interest to forgive debt unilaterally. We first show why debt forgiveness is often not in the creditor's interest.

11.4.1 Unilateral Debt Forgiveness

Consider the situation of a country that owes $100. Assume that there is some uncertainty about whether the country will be able to repay its debt

[3] The analysis that follows draws heavily from a lucid article by Paul Krugman, of Princeton University, entitled "Reducing Developing Country Debt," in *Currencies and Crises,* Paul Krugman (Ed.), Cambridge MA: MIT Press, 1995.

Table 11.5: Initial situation

	Good state	Bad state
Probability of state	$\frac{1}{3}$	$\frac{2}{3}$
Face value = 100		
Receipt of creditors	100	25
Expected repayment: 50		
Secondary market price:0.50		

in full. In particular, suppose that there are two possible outcomes (see table 11.5). Either the country will be able to repay its debt in full, we refer to this scenario as the good state. Or it will only be able to pay 25, we call this the bad state. Suppose that the good state occurs with probability 1/3 (so that bad state occurs with probability 2/3). Thus,

$$\text{expected repayment to creditors} = 100 \times 1/3 + 25 \times 2/3 = 50.$$

This means that the country's debt, whose face value is 100, is indeed worth only 50. The price of each unit of debt in the secondary market is accordingly only 0.50:

$$\text{secondary market price} \;=\frac{\text{Expected repayment}}{\text{Face value of the debt}} = \frac{50}{100} = 0.50$$

Suppose now that the creditors forgive 50 units of debt. Then the remaining debt outstanding is only 50 ($D = 50$). What is the new secondary market price? As shown in table 11.6, in the bad state the country can again only pay 25 but in the good state it will pay the face value of the debt, which, after the debt reduction, is 50. Expected receipts of the creditors then are: $50 \times 1/3 + 25 \times 2/3 = 33.33$. The secondary market price rises to

Table 11.6: Unilateral debt forgiveness of 50

	Good state	Bad state
Probability of state	$\frac{1}{3}$	$\frac{2}{3}$
D = 50		
Receipt of creditors	50	25
Expected repayment: =33.33		
Secondary market price:=0.67		

$33.33/50 = .67$. The loss from debt forgiveness to creditors is the difference between the expected repayment without debt forgiveness, 50, and the expected repayment with debt forgiveness, 33.33, that is, 16.67. Clearly, in this example creditors will never agree to debt forgiveness. The problem is that in this situation, debt forgiveness does not inprove the debtor's capacity to pay in the bad state. It simply makes the debtor country's life easy in the good state, which is precicely the one in which it can afford to pay back.

11.4.2 Debt Overhang

However, in reality creditors sometimes do agree to forgive debt. For example, at the G-7 Economic Summit held in Cologne, Germany in June 1999, rich countries launched a program, dubbed the Cologne Initiative, aimed at reducing the debt burden of the so-called Highly Indebted Poor Countries (HIPCs).[4] To understand why it can be in the creditor's interest to forgive debt, it is important to note that one unrealistic assumption of the above example is that the ability of the debtor to pay is independent of the size of his

[4]For more information on ongoing efforts to reduce the debt burden of HIPCs see the web site of the Center for International Development at Harvard University (http://www.cid.harvard.edu/cidhipc/hipchome.htm).

debt obligations. There are reasons to believe that debtors are more likely to default on their debts the larger is the face value of debt. One reason why this is so is that if D is very large, then the benefits of efforts to improve the economic situation in the debtor country mainly go to the creditors (in the form of large debt-service-related outflows), giving the debtor country very little incentives to improve its economic fundamentals. Another reason why debt repudiation might become more likely as the level of debt gets high is that the debt burden might ultimately appear as a tax on domestic capital implicit in the government's need to collect large amounts of resources to meet external obligations, and thus act as a disincentive for domestic investment. The idea that the probability of repayment is low when the level of debt is high has come to be known as the *debt overhang argument.*

We can formalize the debt overhang argument as follows. Let π be the probability that the good state occurs. Assume that π depends negatively on D:

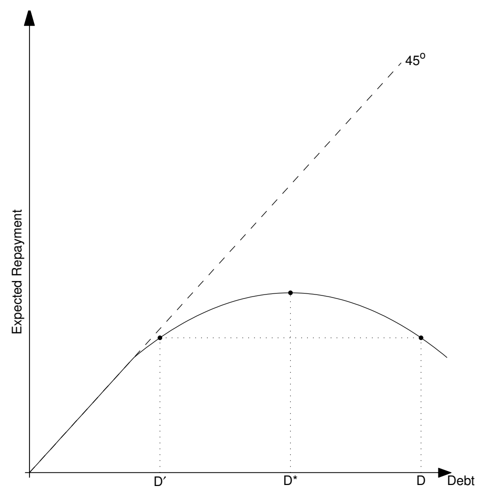
$$\pi = \pi(D); \quad \frac{d\pi(D)}{dD} < 0$$

Assume, as in our original example, that in the bad state the country pays only 25 while in the good state it pays the debt in full. Let D denote the face value of the contry's outstanding debt, and assume that $D > 25$. Then, expected receipts of the creditor are given by

$$\text{expected repayment} = \pi(D) \times D + (1 - \pi(D)) \times 25.$$

Is it still the case that expected repayment is decreasing in the amount of debt forgiven? The answer is no, not necessarily. If an increase in debt

Figure 11.5: The debt Laffer curve



pushes up the probability of the bad state sufficiently, then it can be the case that expected receipts actually fall as D increases. Figure 11.5 shows the relationship between the magnitude of debt outstanding and expected receipts of creditors, also known as *the debt Laffer curve.* Expected repayment peaks at a value of debt equal to D^*. The creditor of a country with an outstanding debt equal to D, for example, can increase his expected receipts by forgiving debt in any amount less than $D - D'$. In particular, the creditor will maximize expected repayment by forgiving $D - D^*$ units of debt. Note that the optimal amount of debt relief does not result in a secondary market price of unity. In the figure, the secondary market price is given by the ratio of the debt Laffer curve to the 45 degree line. The secondary market price becomes unity only if the creditor accepts to reduce the debt to 25, for in this case the risk of default disappears.

Let's illustrate the concept of debt overhang by means of a numerical example. Consider again the case shown in table 11.5. Suppose now creditors forgive 20 of the outstanding debt, so that the new amount of debt is 80. Assume also that this reduction in the debt burden increases the probability of the good state from 1/3 to 1/2. Expected repayments are then given by $80 \times 1/2 + 25 \times 1/2 = 52.5$. Thus expected repayments increase by 2.5 even though the face value of the debt fell by 20. Creditors would benefit from such a unilateral debt reduction. Debtors would also benefit because in case the good state occurs, they have to pay 20 less than in the absence of the debt reduction scheme. To sum up, if a country is on the "wrong" (downward sloping) side of the debt Laffer curve, then it will be the case that unilateral debt forgiveness is not necessarily against the interest of creditors. Thus, one should not be surprised to see debt forgiveness happen sometimes.

11.4.3 The Free Rider Problem In Debt Forgiveness

Even in the case that unilateral debt forgiveness benefits the creditors, in practice, such schemes might be difficult to implement. The reason is that they create a "free rider" problem. Going back to the above example, suppose that only some of the creditors forgive debt but others choose not to participate. As a result of the debt forgiveness, the secondary market price of debt increases from 0.5 to $52.5/80 = .66$ benefiting those who chose not to participate in the scheme. So, from the point of view of an individual creditor it is always best not to forgive any debt and hope that some of the other creditors do and then free ride on the debt reduction efforts of other creditors. Because of this free rider problem, if debt forgiveness occurs in practice it is usually a *concerted* effort, namely one where *all* creditors agree on forgiving some part of the debt.

11.4.4 Third-party debt buy-backs

A debt-reduction scheme often considered by multinational organizations is third-party debt buy backs. A third-party debt buy-back consists in purchases of developing country debt at secondary market prices by a third party, such as the World Bank, the Inter American Development Bank, or the International Monetary Fund, with the purpose of reducing the debt burden of such countries. The third party buy some external debt in the secondary market and immediately forgives that debt (i.e., destrys the pieces of paper it bought).

Consider our original numerical example of a country that has an outstanding debt of 100; the country can pay 100 in the good state and only 25 in the bad state. The good state occurs with probability 1/3 and the bad state with probability 2/3. The secondary market price of debt is 0.50 and expected payments are 50.

Suppose now that the World Bank announces that it will buy 75 units of (face value) debt in the secondary market. As soon as the announcement is made, the secondary market price jumps to a new value. Specifically, after the buy back the level of outstanding debt is 25, which the debtor country can pay in any state, good or bad. Thus, expected payments are 25, which is also the face value of the remaining outstanding debt. This implies that the secondary market price jumps up from 0.50 to 1 at the announcement of the buy-back and before it actually takes place. Who benefits from the buy-back? Creditors receive 75 from the World Bank and 25 from the debtor country. Thus, comparing the situation with and without buy-back, creditors benefit from the buy-back by 50, because in the absence of the buy-back scheme their expected receipts were 50 whereas after the buy-back they are 100. Debtors have expected payments of 50 in the absence of the debt-reduction scheme and 25 when the debt buy-back is in place. So they benefit by 25. Summing up, the World Bank pays 75, of which 50 go to the creditors and 25 to the debtor countries.

We conclude that this method of introducing debt relief is expensive—the World Bank ends up paying par value for the debt it buys back—and benefits mostly the creditors rather than the debtors whom the World Bank meant to help.

11.4.5 Debt swaps

Another type of debt reduction scheme is given by debt swaps. A debt swap consist in the issuance of new debt with seniority over the old debt. The new debt is then used to retire old debt. It is important that the new debt be made senior to the existing debt. This means that at the time of servicing and paying the debt, the new debt is served first.

Consider again the original numerical example described in table 11.5. The debtor country pays the face value of the debt, 100, with probability 1/3 and 25 with probability 2/3. Thus, expected payments are 50 and the secondary market price is 0.5. Suppose now that the government issues 25 units of new debt with the characteristic that the new debt has seniority over the old debt. The new debt is default free. To see this, note that in the bad state the government has 25, which suffices to pay back the new debt. This implies that the debtor government is able to introduce the new debt at par, i.e., the price of new debt is unity. At the same time, because in the bad state all of the debtor resources are devoted to paying back the new debt, the government defaults on the totality of the outstanding old debt if the state of nature turns out to be bad. Let D^o denote the outstanding stock of old debt after the swap. Holders of this debt receive payments in the amount D^o in the good state and 0 in the bad state. So expected payments on the outstanding old debt equal $1/3 \times D^o + 2/3 \times 0 = 1/3 \times D^o$. The secondary market price of the outstanding old debt is the ratio of the expected payments to the face value, or $(1/3 \times D^o)/D^o = 1/3$. Notice that the price of old debt experiences a sharp decline from 0.5 to 0.33. At this price, the government can use the 25 dollars raised by floating new debt to retire, or swap, $25/0.33 = 75$ units of old debt. As a result, after the swap the outstanding amount of old debt falls from 100 to 25, that is, $D^o = 25$.

Who benefits from this swap operation? Clearly the debtor country. In the absence of a swap, the debtor has expected payments of 50. With the swap, the debtor has expected payments of 8.33 to holders of old debt and 25 to holders of new debt. These two payments add up to only 33.33. So the government gains 16.67=50-33.33 by implementing the swap. On the other hand, creditors see their receipts fall from 50 before the swap to 33.33 after the swap (25 from the new debt and 8.33 from the old debt).

The Greek Debt Swap of March 2012

A recent example of a debt swap is the restructuring of Greek government debt that took place in the aftermath of the 2008 worldwide recession, which had thrown Greece into a particularly severe economic crisis. By 2011, GDP was falling at a rate of 7.5 percent per year, and unemployment was soaring reaching around 25 percent (50 percent among young workers). By March 2012 it had become clear that the Greek government could no longer service its debt, which exceeded 170 percent of GDP and made Greece the most highly indebted sovereign in the European Union. The face value of Greek government debt outstanding at the time was around €350 billion, of which €206 billion were held by private creditors and the rest by foreign governments and international institutions such as the European Central Bank and the International Monetary Fund. The Greek government proposed a debt swap for privately held debt that included three key elements: a write down of the face value, a reduction in interest rates, and an increase in maturities. The debt swap took the form of an exchange of €465 of new bonds for each €1,000 of old bonds outstanding. The €465 of new debt was composed of €150 of bonds issued and guaranteed by the European Financial Stability Facility and €315 of bonds issued by the Greek government. The new bonds would start paying interest for the first time in 2023, greatly reducing the pressures on the Greek fiscal deficit over the short and medium run. At the same time, Greece committed to continue to service its debt held by foreign governments and international institutions. In fact, a new law was enacted according to which any tax revenue must first be used to service the debt before it could fund any other government expenditures.

The vast majority of the privately held old debt, €177 billion, was issued under Greek law, and the remainder, €29 billion under foreign law. The Greek government passed a law that bound all private bond holders of debt issued under Greek law to the bond-swap if more than two-thirds of them consented to it. Faced with the alternative of outright default by Greece, most private creditors quickly agreed to the swap and thus the debt-swap was applied to the entire €177 billion of debt outstanding issued under Greek law. In addition, private holders of €20 billion of Greek government bonds issued under foreign law also chose to participate in the debt swap, so that in the end of the €206 billion of Greek debt held by the private sector, €197 billion was exchanged for new debt with a face value of €92 billion. That is, the debt swap resulted in a debt write down of €105 billion and €9 billion remain in the hands of opportunistic holdouts.

11.5 Exercises

1. External Debt Restructuring

   Consider an economy with an external debt of D. Assume that the economy's capacity to honor its debt is state dependent. Specifically, suppose that there are 2 states, denoted good and bad. In the good state the country can pay its debt in full. In the bad state the country can at most pay 20. The probability of the bad state, which we will denote by $\pi(D)$, is given by $\pi(D) = 0.01D$.

   (a) Debt Forgiveness

   Suppose that the country's debt is 80. What is the secondary market price of debt? Would it be in creditors collective interest to forgive 10 units of debt?

   (b) A Third-Party Debt Buy-Back

   Suppose now that the country's debt is 50 and the a debt relief agency agrees to buy back 10 units of debt in the secondary market. (The debt relief agency will then forgive the debt it holds.) What price will the agency have to pay for each unit of debt it buys back? What is the total cost of the operation? By how much does the expected income of creditors increase? By how much do the expected payments of the debtor country decline?

2. A Debt Swap

   Suppose a country has 100 units of debt outstanding. In the good state of the world the country can repay its debts in full, in the bad state of the world, the country can repay only 40. The good and the bad states occur with equal probability.

(a) Calculate the repayments the creditors are expecting to receive.

(b) Find the secondary market price of one unit of debt.

Now assume that the country wants to restructure its debt. It announces that it will introduce two new securities, of type A and B, respectively, to replace the 100 old securities outstanding by means of a debt swap. It issues 40 units of securities of type A, which will pay in full in the good and the bad states, and 20 units of securities of type B, which pay in full only in the good state of the world. Both new securities are senior to the old existing debt.

(c) At what rate can the government swap old securities for new securities of type A.

(d) At what rate can the government swap old securities for new securities of type B.

(e) What is the net effect of this debt restructuring on the expected repayments to creditors.

Chapter 12

Monetary Policy and Nominal Exchange Rate Determination

Thus far, we have focused on the determination of *real* variables, such as consumption, the trade balance, the current account, and the real exchange rate. In this chapter, we study the determination of *nominal* variables, such as the nominal exchange rate, the price level, inflation, and the quantity of money.

We will organize ideas around using a theoretical framework (model) that is similar to the one presented in previous chapters, with one important modification: there is a demand for money.

An important question in macroeconomics is why households voluntarily choose to hold money. In the modern world, this question arises because money takes the form of unbacked paper notes printed by the government. This kind of money, one that the government is not obliged to exchange for goods, is called fiat money. Clearly, fiat money is intrinsically valueless. One reason why people value money is that it facilitates transactions. In the absence of money, all purchases of goods must take the form of barter. Barter exchanges can be very difficult to arrange because they require double coincident of wants. For example, a carpenter who wants to eat an ice cream must find an ice cream maker that is in need of a carpenter. Money eliminates the need for double coincidence of wants. In this chapter we assume that agents voluntarily hold money because it facilitates transactions.

12.1 The quantity theory of money

What determines the level of the nominal exchange rate? Why has the Euro been depreciating vis-a-vis the US dollar since its inception in 1999? The quantity theory of money asserts that a key determinant of the exchange rate is the quantity of money printed by central banks.

According to the quantity theory of money, people hold a more or less stable fraction of their income in the form of money. Formally, letting Y denote real income, M^d money holdings, and P the price level (i.e., the price of a representative basket of goods), then

$$M^d = \kappa P \cdot Y$$

This means that the *real* value of money, M^d/P, is determined by the level of real activity of the economy. Let $m^d \equiv M^d/P$ denote the demand for real money balances. The quantity theory of money then maintains that m^d is determined by nonmonetary or real factors such as aggregate output, the degree of technological advancement, etc.. Let M^s denote the nominal money *supply*, that is, M^s represents the quantity of bills and coins in circulation plus checking deposits. Equilibrium in the money market requires that money demand be equal to money supply, that is,

$$\frac{M^s}{P} = m^d \tag{12.1}$$

A similar equilibrium condition has to hold in the foreign country. Let M^{*s} denote the foreign nominal money supply, P^* the foreign price level, and m^{*d} the demand for real balances in the foreign country. Then,

$$\frac{M^{*s}}{P^*} = m^{*d} \tag{12.2}$$

Let E denote the nominal exchange rate, defined as the domestic-currency price of the foreign currency. So, for example, if E refers to the dollar/euro exchange rate, then stands for the number of US dollars necessary to purchase one euro. Let e denote the real exchange rate. As explained in previous chapters, e represents the relative price of a foreign basket of goods in terms of domestic baskets of goods. Formally,

$$e = \frac{E\,P^*}{P}$$

Using this expression along with (12.1) and (12.2), we can express the nominal exchange rate, E, as

$$E = \frac{M}{M^*}\left(\frac{e\ m^*}{m}\right) \tag{12.3}$$

According to the quantity theory of money, not only m and m^* but also e are determined by non-monetary factors. The quantity of money, in turn, depends on the exchange rate regime maintained by the respective central banks. There are two polar exchange rate arrangements: flexible and fixed exchange rate regimes.

12.1.1 AFloating (or Flexible) Exchange Rate Regime

Under a floating exchange rate regime, the market determines the nominal exchange rate E. In this case the level of the money supplies in the domestic and foreign countries, M^s and M^{*s}, are determined by the respective central banks and are, therefore, exogenous variables. Exogenous variables are those that are determined outside of the model. By contrast, the nominal exchange rate is an endogenous variable in the sense that its equilibrium value is determined within the model.

Suppose, for example, that the domestic central bank decides to increase the money supply M^s. It is clear from equation (12.3) that, all other things constant, the monetary expansion in the home country causes the nominal exchange rate E to depreciate by the same proportion as the increase in the money supply. (i.e., E increases). The intuition behind this effect is simple. An increase in the quantity of money of the domestic country increases the relative scarcity of the foreign currency, thus inducing an increase in the relative price of the foreign currency in terms of the domestic currency, E. In addition, equation (12.1) implies that when M increases the domestic price level, P, increases in the same proportion as M. An increase in the domestic money supply generates inflation in the domestic country. The reason for this increase in prices is that when the central bank injects additional money balances into the economy, households find themselves with more money than they wish to hold. As a result households try to get rid of the excess money balances by purchasing goods. This increase in the demand for goods drives prices up.

Suppose now that the real exchange rate depreciates, (that is e goes up). This means that a foreign basket of goods becomes more expensive relative to a domestic basket of goods. A depreciation of the real exchange rate can be due to a variety of reason, such as a terms-of-trade shock or the removal of import barriers. If the central bank keeps the money supply unchanged, then by equation (12.3) a real exchange rate depreciation causes a depreciation (an increase) of the nominal exchange rate. Note that e and E increase by the same proportion. The price level P is unaffected because neither M nor m have changed (see equation (12.1)).

12.1.2 Fixed Exchange Rate Regime

Under a fixed exchange rate regime, the central bank determines E by intervening in the money market. So given E, M^{*s}, and $e\,m^{*s}/m^s$, equation (12.3) determines what M^s ought to be in equilibrium. Thus, under a fixed exchange rate regime, M^s is an endogenous variable, whereas E is exogenously determined by the central bank.

Suppose that the real exchange rate, e, experiences a depreciation. In this case, the central bank must reduce the money supply (that is, M^s must fall) to compensate for the real exchange rate depreciation. Indeed, the money supply must fall by the same proportion as the real exchange rate. In addition, the domestic price level, P, must also fall by the same proportion as e in order for real balances to stay constant (see equation (12.1)). This implies that we have a deflation, contrary to what happens under a floating exchange rate policy.

12.2 Fiscal deficits, inflation, and the exchange rate

The quantity theory of money provides a simple and insightful analysis of the relationship between money, prices, the nominal exchange rate, and real variables. However, it leaves a number of questions unanswered. For example, what is the effect of fiscal policy on inflation? What role do expectations about future changes in monetary and fiscal policy play for the determination of prices, exchange rates, and real balances? To address these questions, it is necessary to use a richer model. One that incorporates a more realistic money demand specification and that explicitly considers the relationship between monetary and fiscal policy.

In this section, we embed a money demand function into a model with a government sector, similar to the one used in chapter 7, to analyze the effects of fiscal deficits on the current account. Specifically, we consider a small-open endowment economy with free capital mobility, a single traded good per period, and a government that levies lump-sum taxes to finance government purchases. For simplicity, we assume that there is no physical capital and hence no investment. Unlike the models studied thus far, we now assume that the economy exists, not just for 2 periods, but for an infinite number of periods. Such an economy is called an *infinite horizon* economy.

We discuss in detail each of the four building blocks that compose our monetary economy: (1) The demand for money; (2) Purchasing power parity; (3) Interest rate parity; and (4) The government budget constraint.

12.2.1 The Demand For Money

In the quantity theory, the demand for money is assumed to depend only on the level of real activity. In reality, however, the demand for money also depends on the nominal interest rate. In particular, money demand is decreasing in the nominal interest rate. The reason is that money is a non-interest-bearing asset. As a result, the opportunity cost of holding money is the nominal interest rate on alternative interest-bearing liquid assets, such as time deposits, government bonds, and money market mutual funds. Thus, the higher the nominal interest rate the lower is the demand for real money balances. Formally, we assume a money demand function of the form:

$$\frac{M_t}{P_t} = L(\bar{C}, i_t), \tag{12.4}$$

where $\bar{C}$ denotes consumption and i_t denotes the domestic nominal interest rate in period t. The function L is increasing in consumption and decreasing in the nominal interest rate. We assume that consumption is constant over time. Therefore C does not have a time subscript. We indicate that consumption is constant by placing a bar over C. The money demand function $L(\cdot, \cdot)$ is also known as the *liquidity preference function*. Readers interested in learning how a money demand like equation (12.4) can be derived from the optimization problem of the household should consult the appendix to this chapter.

12.2.2 Purchasing power parity (PPP)

Because in the economy under consideration there is a single traded good and no barriers to international trade, purchasing power parity must hold. Let P_t be the domestic currency price of the good in period t, P_t^* the foreign currency price of the good in period t, and E_t the nominal exchange rate in period t, defined as the price of one unit of foreign currency in terms of domestic currency. Then PPP implies that in any period t

$$P_t = E_t P_t^*$$

For simplicity, assume that the foreign currency price of the good is constant and equal to 1 ($P_t^* = 1$ for all t). In this case, it follows from PPP that the domestic price level is equal to the nominal exchange rate,

$$P_t = E_t. \tag{12.5}$$

Using this relationship, we can write the liquidity preference function (12.4) as

$$\frac{M_t}{E_t} = L(\bar{C}, i_t), \tag{12.6}$$

12.2.3 The interest parity condition

In this economy, there is free capital mobility and no uncertainty. Thus, the gross domestic nominal interest rate must be equal to the gross world nominal interest rate times the expected gross rate of devaluation of the domestic currency. This relation is called the *uncovered interest parity condition.* Formally, let E^e_{t+1} denote the nominal exchange rate that agents expect at time t to prevail at time $t+1$, and let i_t denote the domestic nominal interest rate, that is, the rate of return on an asset denominated in domestic currency and held from period t to period $t+1$. Then the uncovered interest parity condition is:

$$1 + i_t = (1 + r^*)\frac{E^e_{t+1}}{E_t} \tag{12.7}$$

In the absence of uncertainty, the nominal exchange rate that will prevail at time $t+1$ is known at time t, so that $E^e_{t+1} = E_{t+1}$. Then, the uncovered interest parity condition becomes

$$1 + i_t = (1 + r^*)\frac{E_{t+1}}{E_t} \tag{12.8}$$

This condition has a very intuitive interpretation. The left hand side is the gross rate of return of investing 1 unit of domestic currency in a domestic currency denominated bond. Because there is free capital mobility, this investment must yield the same return as investing 1 unit of domestic currency in foreign bonds. One unit of domestic currency buys $1/E_t$ units of the foreign bond. In turn, $1/E_t$ units of the foreign bond pay $(1+r^*)/E_t$ units of foreign currency in period $t+1$, which can then be exchanged for $(1+r^*)E_{t+1}/E_t$ units of domestic currency.[1]

12.2.4 The government budget constraint

The government has three sources of income: real tax revenues, T_t, money creation, $M_t - M_{t-1}$, and interest earnings from holdings of international bonds, $E_t r^* B^g_{t-1}$, where B^g_{t-1} denotes the government's holdings of foreign currency denominated bonds carried over from period $t-1$ into period t and r^* is the international interest rate. Government bonds, B^g_t, are denominated in foreign currency and pay the world interest rate r^*. The government allocates its income to finance government purchases, $P_t G_t$, where G_t denotes real government consumption of goods in period t, and to changes in its holdings of foreign bonds, $E_t(B^g_t - B^g_{t-1})$. Thus, in period t, the government budget constraint is

$$E_t(B^g_t - B^g_{t-1}) + P_t G_t = P_t T_t + (M_t - M_{t-1}) + E_t r^* B^g_{t-1}$$

[1]Here two comments are in order. First, in chapter 8, we argued that free capital mobility implies that *covered* interest rate parity holds. The difference between covered and uncovered interest rate parity is that covered interest rate parity uses the forward exchange rate F_t to eliminate foreign exchange rate risk, whereas uncovered interest rate parity uses the expected future spot exchange rate, E^e_{t+1}. In general, F_t and E^e_{t+1} are not equal to each other. However, under certainty $F_t = E^e_{t+1} = E_{t+1}$, so covered and uncovered interest parity are equivalent. Second, in chapter 8 we further argued that free capital mobility implies that covered interest parity must hold for *nominal* interest rates. However, in equation (12.7) we used the world *real* interest rate r^*. In the context of our model this is okay because we are assuming that the foreign price level is constant ($P^* = 1$) so that, by the Fisher equation (8.2), the nominal world interest rate must be equal to the real world interest rate ($i^*_t = r^*_t$).

The left hand side of this expression represents the government's uses of revenue and the right hand side the sources. Note that B_t^g is not restricted to be positive. If B_t^g is positive, then the government is a creditor, whereas if it is negative, then the government is a debtor.[2] We can express the government budget constraint in real terms by dividing the left and right hand sides of the above equation by the price level P_t. Using the result that $E_t = P_t$, and after rearranging terms, we have

$$B_t^g - B_{t-1}^g = \frac{M_t - M_{t-1}}{P_t} - \left[G_t - T_t - r^* B_{t-1}^g\right] \tag{12.9}$$

The first term on the right hand side measures the government's real revenue from money creation and is called *seignorage revenue*,

$$\text{seignorage revenue} = \frac{M_t - M_{t-1}}{P_t}.$$

The second term on the right hand side of (12.9) is the *secondary fiscal deficit* and we will denote it by DEF_t. Recall from chapter 7 that the secondary fiscal deficit is given by the difference between government expenditures and income from the collection of taxes and interest income from bond holdings. Formally, DEF_t is defined as

$$DEF_t = (G_t - T_t) - r^* B_{t-1}^g$$

[2]Note that the notation here is different from the one used in chapter 7, where B_t^g denoted the level of government debt.

In chapter 7, we also defined the primary fiscal deficit as the difference between government expenditures and tax revenues $(G_t - T_t)$, so that the secondary fiscal deficit equals the difference between the primary fiscal deficit and interest income from government holdings of interest bearing assets.

Using the definition of secondary fiscal deficit and the fact that by PPP $P_t = E_t$, the government budget constraint can be written as

$$B_t^g - B_{t-1}^g = \frac{M_t - M_{t-1}}{E_t} - DEF_t \tag{12.10}$$

This equation makes it transparent that a fiscal deficit $(DEF_t > 0)$ must be associated with money creation $(M_t - M_{t-1} > 0)$ or with a decline in the government's asset position $(B_t^g - B_{t-1}^g < 0)$, or both. To complete the description of the economy, we must specify the exchange rate regime, to which we turn next.

12.3 A fixed exchange rate regime

Under a fixed exchange rate regime, the government intervenes in the foreign exchange market in order to keep the exchange rate at a fixed level. Let that fixed level be denoted by E. Then $E_t = E$ for all t. When the government pegs the exchange rate, the money supply becomes an endogenous variable because the central bank must stand ready to exchange domestic for foreign currency at the fixed rate E. With the nominal exchange rate E, the PPP condition, given by equation (12.5), implies that the price level, P_t, is also constant and equal to E for all t. Because the nominal exchange rate is constant, the expected rate of devaluation is zero. This implies, by the interest parity condition (12.8), that the domestic nominal interest rate, i_t, is constant and equal to the world interest rate r^*. It then follows from the liquidity preference equation (12.6) that the demand for nominal balances is constant and equal to $EL(\bar{C}, r^*)$. Since in equilibrium money demand must equal money supply, we have that the money supply is also constant over time: $M_t = M_{t-1} = EL(\bar{C}, r^*)$. Using the fact that the money supply is constant, the government budget constraint (12.10) becomes

$$B_t^g - B_{t-1}^g = -DEF_t \tag{12.11}$$

In words, when the government pegs the exchange rate, it loses one source of revenue, namely, seignorage. Therefore, fiscal deficits must be entirely financed through the sale of interest bearing assets.

12.3.1 Fiscal deficits and the sustainability of currency pegs

For a fixed exchange rate regime to be sustainable over time, it is necessary that the government displays fiscal discipline. To see this, suppose that the government runs a constant secondary fiscal deficit, say $DEF_t = DEF > 0$ for all t. Equation (12.11) then implies that government assets are falling over time ($B_t^g - B_{t-1}^g = -DEF < 0$). At some point B_t^g will become negative, which implies that the government is a debtor. Suppose that there is an upper limit on the size of the public debt. Clearly, when the public debt hits this limit, the government is forced to eliminate the fiscal deficit (i.e., set $DEF = 0$), or default on its debt (as Greece did in 2012), or abandon the exchange rate peg. The latter alternative is called a *balance of payments crisis*. We will analyze balance of payments crises in more detail in section 12.6.

The fiscal consequences of a devaluation

Consider now the effects of a once-and-for-all devaluation of the domestic currency. We will show that this policy is equivalent to a lump-sum tax . To see this, assume that in period 1 the government unexpectedly announces an increase in the nominal exchange rate from E to $E' > E$, that is, $E_t = E'$ for all $t \geq 1$. By the PPP condition, equation (12.5), the domestic price level, P_t, jumps up in period 1 from E to E' and remains at that level thereafter.

By the interest rate parity condition (12.8), we have that the nominal interest rate in period 1 is given by

$$1 + i_t = (1 + r^*)\frac{E_2}{E_1} == (1 + r^*)\frac{E'}{E'} = (1 + r^*).$$

Because the nominal interest rate was equal to r^* before period 1, it follows that an unexpected, once-and-for-all devaluation has no effect on the domestic nominal interest rate. The reason why the nominal interest rate remains unchanged is that it depends on the *expected future* rather than the *current* rate of devaluation. In period 0, households did not expect the government to devalue the domestic currency in period 1. Therefore, the expected devaluation rate in period 0 was zero and the nominal interest rate was equal to r^*. In period 1, households expect no further devaluations of the domestic currency in period 2, therefore the nominal interest rate is also equal to r^* in period 1. Similarly, because agents expect the nominal exchange rate to be constant and equal to E' for all future periods, the expected rate of devaluation is nil and the nominal interest rate equals r^* forever.

Using the fact that the nominal interest rate is unchanged, the liquidity preference equation (12.6) then implies that in period 1 the demand for nominal money balances increases from $EL(\bar{C}, r^*)$ to $E'L(\bar{C}, r^*)$. This means that the demand for nominal balances must increase by the same proportion as the nominal exchange rate. Consider now the government budget constraint in period 1.

$$\begin{aligned} B_1^g - B_0^g &= \frac{M_1 - M_0}{E'} - DEF \\ &= \frac{E'L(\bar{C}, r^*) - EL(\bar{C}, r^*)}{E'} - DEF \end{aligned}$$

The numerator of the first term on the right-hand side of the last equality is clearly positive, since $E' > E$. Therefore, in period 1 seignorage revenue is positive. In the absence of a devaluation, seignorage revenue would have been nil because in that case $M_1 - M_0 = EL(\bar{C}, r^*) - EL(\bar{C}, r^*) = 0$. Therefore, a devaluation increases government revenue in the period in which the devaluation takes place. In the periods after the devaluation, $t = 2, 3, 4, \ldots$, the nominal money demand is constant and equal to $E'L(\bar{C}, r^*)$, so that $M_t - M_{t-1} = 0$ for all $t \geq 2$ and seignorage revenue is nil.

Summarizing, by PPP, a devaluation produces an increase in the domestic price level of the same proportion as the increase in the nominal exchange rate. Given the households' holdings of nominal money balances the increase in the price level implies that real balances will decline. Thus, a devaluation acts as a tax on real balances. In order to rebuild their desired real balances, which don't change because the nominal interest rate is unaffected by the devaluation, households will sell part of their foreign bonds to the central bank in return for domestic currency. The net effect of a devaluation is, therefore, that the private sector ends up with a lower foreign asset position but the same level of real balances, whereas the government gains real resources as it exchanges money created by itself for interest-bearing foreign assets.

12.4 A constant-money-growth-rate regime

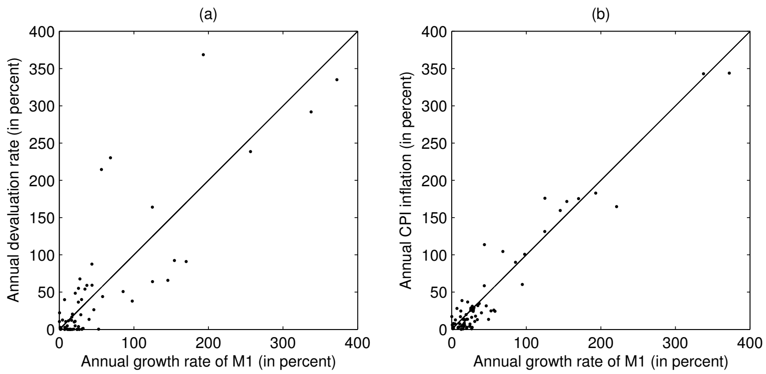
We now consider a monetary policy regime in which the central bank targets a certain path for the money supply and does not directly target a path for the nominal exchange rate. For this reason, we say that the central bank lets the nominal exchange *float*. The monetary/exchange rate regime studied here is exactly the opposite to the one studied in subsection 12.3, where the central bank fixed the nominal exchange rate and let the quantity of money be market (or endogenously) determined.

Consider a specific target for the path of the money supply in which the central bank expands the quantity of money at a constant, positive rate μ each period, so that

$$M_t = (1 + \mu)M_{t-1} \tag{12.12}$$

Our goal is to find out how the endogenous variables of the model, such as the nominal exchange rate, the price level, real balances, the domestic

Figure 12.1: Devaluation, inflation, and money growth. Argentina 1901-2005



nominal interest rate, and so forth behave under the monetary/exchange rate regime specified by equation (12.12). To do this, we will conjecture (or guess) that in equilibrium the nominal exchange rate depreciates at the rate μ. We will then verify that our guess is correct. Thus, we are guessing that

$$\frac{E_{t+1}}{E_t} = 1 + \mu,$$

for $t = 1, 2, \ldots$. Because PPP holds and the foreign price level is one (i.e., $P_t = E_t$), the domestic price level must also grow at the rate of monetary expansion μ,

$$\frac{P_{t+1}}{P_t} = 1 + \mu.$$

for $t = 1, 2, \ldots$. This expression says that, given our guess, the rate of inflation must equal the rate of growth of the money supply. Panels (a) and (b) of figure 12.1 display annual averages of the rate of depreciation of the Argentine currency vis-à-vis the U.S. dollar, the Argentine money growth rate, and the Argentine inflation rate for the period 1901-2005. (We omitted the years 1984, 1985, 1989, 1990 where annual money growth rates exceeded 400 perent.) The data is roughly consistent with the model in showing that there exists a close positive relationship between these three variables.[3]

To determine the domestic nominal interest rate i_t, use the interest parity condition (12.8)

$$1 + i_t = (1 + r^*)\frac{E_{t+1}}{E_t} = (1 + r^*)(1 + \mu),$$

which implies that the nominal interest rate is constant and increasing in μ. When μ is positive, the domestic nominal interest rate exceeds the real interest rate r^* because the domestic currency is depreciating over time. We summarize the positive relationship between i_t and μ by writing

$$i_t = i(\mu)$$

The notation $i(\mu)$ simply indicates that i_t is a function of μ. The function $i(\mu)$ is increasing in μ. Substituting this expression into the liquidity preference function (12.6) yields

$$\frac{M_t}{E_t} = L(\bar{C}, i(\mu)). \tag{12.13}$$

Note that $\bar{C}$ is a constant and that because the money growth rate μ is

[3]Strictly speaking, the model predicts that all points in both figures should lie on a straight line, which is clearly not the case. The reason for this discrepancy may be that the model abstracts from a number of real world factors that affect the relationship between money growth, inflation, and depreciation. For example, in the model we assume that there is no domestic growth, that all goods are traded, that PPP holds, and that foreign inflation is constant.

constant, the nominal interest rate $i(\mu)$ is also constant. Therefore, the right hand side of (12.13) is constant. For the money market to be in equilibrium, the left-hand side of (12.13) must also be constant. This will be the case only if the exchange rate depreciates—grows—at the same rate as the money supply. This is indeed true under our initial conjecture that $E_{t+1}/E_t = 1 + \mu$. Equation (12.13) says that in equilibrium real money balances must be constant and that the higher the money growth rate μ the lower the equilibrium level of real balances.

12.5 The Inflation Tax

Let's now return to the government budget constraint (12.10), which we reproduce below for convenience

$$B_t^g - B_{t-1}^g = \frac{M_t - M_{t-1}}{E_t} - DEF_t$$

Let's analyze the first term on the right-hand side of this expression, seignorage revenue. Using the fact that $M_t = E_t L(\bar{C}, i(\mu))$ (equation (12.13)), we can write

$$\begin{aligned}\frac{M_t - M_{t-1}}{E_t} &= \frac{E_t L(\bar{C}, i(\mu)) - E_{t-1} L(\bar{C}, i(\mu))}{E_t} \\ &= L(\bar{C}, i(\mu)) \left(\frac{E_t - E_{t-1}}{E_t} \right)\end{aligned}$$

Using the fact that the nominal exchange rate depreciates at the rate μ, that is, $E_t = (1+\mu)E_{t-1}$, to eliminate E_t and E_{t-1} from the above expression, we can write seignorage revenue as

$$\frac{M_t - M_{t-1}}{E_t} = L(\bar{C}, i(\mu)) \left(\frac{\mu}{1+\mu}\right) \tag{12.14}$$

Thus, seignorage revenue is equal to the product of real balances, $L(\bar{C}, i(\mu))$, and the factor $\mu/(1+\mu)$.

The right hand side of equation (12.14) can also be interpreted as the *inflation tax*. The idea is that inflation acts as a tax on the public's holdings of real money balances. To see this, let's compute the change in the real value of money holdings from period $t-1$ to period t. In period $t-1$ nominal money holdings are M_{t-1} which have a real value of M_{t-1}/P_{t-1}. In period t the real value of M_{t-1} is M_{t-1}/P_t. Therefore we have that the inflation tax equals $M_{t-1}/P_{t-1} - M_{t-1}/P_t$, or, equivalently,

$$\text{inflation tax} = \frac{M_{t-1}}{P_{t-1}} \frac{P_t - P_{t-1}}{P_t}$$

where M_{t-1}/P_{t-1} is the tax base and $(P_t - P_{t-1})/P_t$ is the tax rate. Using the facts that in our model real balances are equal to $L(\bar{C}, i(\mu))$ and that $P_t/P_{t-1} = 1+\mu$, the inflation tax can be written as

$$\text{inflation tax} = L(\bar{C}, i(\mu)) \frac{\mu}{1+\mu},$$

which equals seignorage revenue. In general seignorage revenue and the inflation tax are not equal to each other. They are equal in the special case that real balances are constant over time, like in our model when the money supply expands at a constant rate.

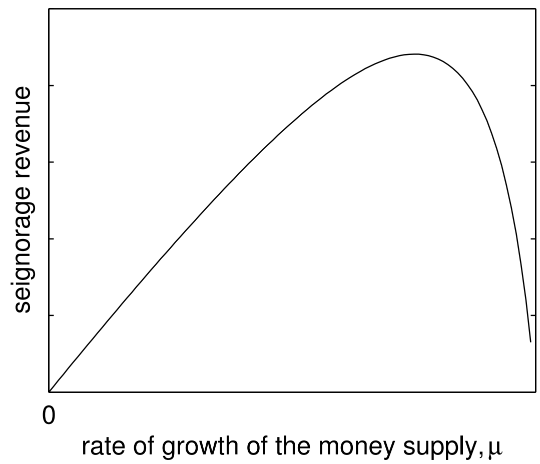
12.5.1 The Inflation Tax Laffer Curve

Because the tax base, i.e., real balances, is decreasing in μ and the tax rate, $\mu/(1+\mu)$, is increasing in μ, it is not clear whether seignorage increases or decreases with the rate of expansion of the money supply. Whether seignorage revenue is increasing or decreasing in μ depends on the form of the liquidity preference function $L(\cdot,\cdot)$ as well as on the level of μ itself. Typically, for low values of μ seignorage revenue is increasing in μ. However, as μ gets large the contraction in the tax base (the money demand) dominates the increase in the tax rate and therefore seignorage revenue falls as μ increases. Thus, there exists a *maximum level of revenue* a government can collect from printing money. The resulting relationship between the growth rate of the money supply and seignorage revenue has the shape of an inverted-U and is called the *inflation tax Laffer curve* (see figure 12.2).

12.5.2 Inflationary finance

We now use the theoretical framework developed thus far to analyze the link between fiscal deficits, prices, and the exchange rate. Consider a situation in which the government is running constant fiscal deficits $DEF_t = DEF > 0$ for all t. Furthermore, assume that the government has reached its borrowing limit and thus cannot finance the fiscal deficits by issuing additional debt, so that $B_t^g - B_{t-1}^g$ must be equal to zero. Under these circumstances,

Figure 12.2: The Laffer curve of inflation



the government budget constraint (12.10) becomes

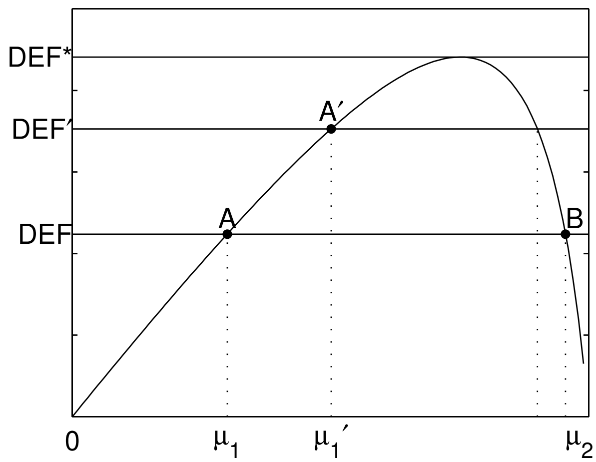
$$DEF = \frac{M_t - M_{t-1}}{E_t}$$

It is clear from this expression, that a country that has exhausted its ability to issue public debt must resort to printing money in order to finance the fiscal deficit. This way of financing the public sector is called *monetization of the fiscal deficit.* Combining the above expression with (12.14) we obtain

$$DEF = L(\bar{C}, i(\mu)) \left(\frac{\mu}{1+\mu} \right). \tag{12.15}$$

Figure 12.3 illustrates the relationship between fiscal deficits and the rate of monetary expansion implied by this equation. The Laffer curve of inflation corresponds to the right hand side of (12.15). The horizontal line plots the left hand side (12.15), or DEF. There are two rates of monetary expansion,

Figure 12.3: Inflationary finance and the Laffer curve of inflation



μ_1 and μ_2, that generate enough seignorage revenue to finance the fiscal deficit DEF. Thus, there exist two equilibrium levels of monetary expansion associated with a fiscal deficit equal to DEF. In the μ_2 equilibrium, point B in the figure, the rates of inflation and of exchange rate depreciation are relatively high and equal to μ_2, whereas in the μ_1 equilibrium, point A in the figure, the rates of inflation and depreciation are lower and equal to μ_1. Empirical studies show that in reality, economies tend to be located on the upward sloping branch of the Laffer curve. Thus, the more realistic scenario is described by point A.

Consider now the effect of an increase in the fiscal deficit from DEF to $DEF' > DEF$. To finance the larger fiscal deficit, the government is forced to increase the money supply at a faster rater. At the new equilibrium, point A', the rate of monetary expansion, ${\mu_1}'$ is greater than at the old equilibrium. As a result, the inflation rate, the rate of depreciation of the domestic currency, and the nominal interest rate are all higher.

The following numerical example provides additional insight on the connection between money creation and fiscal deficits. Suppose that the liquidity preference function is given by:

$$\frac{M_t}{E_t} = \gamma \bar{C} \left(\frac{1 + i_t}{i_t} \right)$$

Suppose that the government runs a fiscal deficit of 10% of GDP ($DEF/Q = 0.1$), that the share of consumption in GDP is 65% ($\bar{C}/Q = 0.65$), that the world real interest rate is 5% per year ($r^* = 0.05$), and that γ is equal to 0.2. The question is what is the rate of monetary expansion necessary to monetize the fiscal deficit. Combining equations (12.5.2) and (12.15) and using the fact $1 + i_t = (1 + r^*)(1 + \mu)$ we have,

$$DEF = \gamma \bar{C} \frac{(1 + r^*)(1 + \mu)}{(1 + r^*)(1 + \mu) - 1} \frac{\mu}{1 + \mu}$$

Divide the left and right hand sides of this expression by Q and solve for μ to obtain

$$\mu = \frac{r^*(DEF/Q)}{(1 + r^*)(\gamma(\bar{C}/Q) - (DEF/Q))} = \frac{0.05 \times 0.1}{1.05 \times (0.2 \times 0.65 - 0.1)} = 0.16$$

The government must increase the money supply at a rate of 16% per year. This implies that both the rates of inflation and depreciation of the domestic currency in this economy will be 16% per year. The nominal interest rate is 21% per year. At a deficit of 10% of GDP, the Laffer curve is rather flat. For example, if the government cuts the fiscal deficit by 1% of GDP, the equilibrium money growth rate falls to 11%.

In some instances, inflationary finance can degenerate into hyperinflation. Perhaps the best-known episode is the German hyperinflation of 1923. Between August 1922 and November 1923, Germany experienced an average monthly inflation rate of 322 percent.[4] More recently, in the late 1980s a number of hyperinflationary episodes took place in Latin America and Eastern Europe. One of the more severe cases was Argentina, where the inflation rate averaged 66 percent per month between May 1989 and March 1990.

A hyperinflationary situation arises when the fiscal deficit reaches a level that can no longer be financed by seignorage revenue alone. In terms of figure 12.3, this is the case when the fiscal deficit is larger than DEF^*, the level of deficit associated with the peak of the Laffer curve. What happens in practice is that the government is initially unaware of the fact that no rate of monetary expansion will suffice to finance the deficit. In its attempt to close the fiscal gap, the government accelerates the rate of money creation. But this measure is counterproductive because the government has entered the downward sloping side of the Laffer curve. The decline in seignorage revenue leads the government to increase the money supply at an even faster rate. These dynamics turn into a vicious cycle that ends in an accelerating inflationary spiral. The most fundamental step in ending hyperinflation is to eliminate the underlying budgetary imbalances that are at the root of the problem. When this type of structural fiscal reforms is undertaken and is understood by the public, hyperinflation typically stops abruptly.

[4] A fascinating account of four Post World War I European hyperinflations is given in Sargent, "The End of Four Big Inflations," in Robert Hall, editor, *Inflation: Causes and Effects,* The University of Chicago Press, Chicago, 1982.

12.5.3 Money growth and inflation in a growing economy

Thus far, we have considered the case in which consumption is constant over time.[5] We now wish to consider the case that consumption is growing over time. Specifically, we will assume that consumption grows at a constant rate $\gamma > 0$, that is,

$$C_{t+1} = (1+\gamma)C_t.$$

We also assume that the liquidity preference function is of the form

$$L(C_t, i_t) = C_t l(i_t)$$

where $l(\cdot)$ is a decreasing function.[6] Consider again the case that the government expands the money supply at a constant rate $\mu > 0$. As before, we find the equilibrium by first guessing the value of the depreciation rate and then verifying that this guess indeed can be supported as an equilibrium outcome. Specifically, we conjecture that the domestic currency depreciates at the rate $(1+\mu)/(1+\gamma) - 1$, that is,

$$\frac{E_{t+1}}{E_t} = \frac{1+\mu}{1+\gamma}$$

Our conjecture says that given the rate of monetary expansion, the higher the rate of economic growth, the lower the rate of depreciation of the domes-

[5] Those familiar with the appendix will recognize that the constancy of consumption is a direct implication of our assumption that the subjective discount rate is equal to the world interest rate, that is, $\beta(1+r^*) = 1$. It is clear from (12.19) that consumption will grow over time only if $\beta(1+r^*)$ is greater than 1.

[6] Can you show that this form of the liquidity preference function obtains when the period utility function is given by $\ln C_t + \theta \ln(M_t/E_t)$. Under this particular preference specification find the growth rate of consumption γ as a function of β and $1+r^*$.

tic currency. In particular, if the government wishes to keep the domestic currency from depreciating, it can do so by setting the rate of monetary expansion at a level no greater than the rate of growth of consumption ($\mu \leq \gamma$). By interest rate parity,

$$\begin{aligned}(1+i_t) &= (1+r^*)\frac{E_{t+1}}{E_t}\\ &= (1+r^*)\frac{(1+\mu)}{(1+\gamma)}\end{aligned}$$

This expression says that the nominal interest rate is constant over time. We can summarize this relationship by writing

$$i_t = i(\mu, \gamma), \quad \text{for all } t$$

where the function $i(\mu, \gamma)$ is increasing in μ and decreasing in γ.

We continue to assume that PPP and that $P_t^* = 1$, which implies that the domestic price level, P_t, must be equal to the nominal exchange rate, E_t. It follows that the domestic rate of inflation must be equal to the rate of depreciation of the nominal exchange rate, that is,

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \frac{E_t - E_{t-1}}{E_{t-1}} = \frac{1+\mu}{1+\gamma} - 1$$

Equilibrium in the money market requires that the real money supply be equal to the demand for real balances, that is,

$$\frac{M_t}{E_t} = C_t l(i(\mu, \gamma)),$$

The right-hand side of this expression is proportional to consumption, and therefore grows at the gross rate $1+\gamma$. The numerator of the left hand side grows at the gross rate $1+\mu$. Therefore, in equilibrium the denominator of the left hand side must expand at the gross rate $(1+\mu)/(1+\gamma)$, which is precisely our conjecture.

Summarizing, when consumption growth is positive, the domestic inflation rate is *lower* than the rate of monetary expansion. The intuition for this result is straightforward. A given increase in the money supply that is not accompanied by an increase in the demand for real balances will translate into a proportional increase in prices. This is because in trying to get rid of their excess nominal money holdings households attempt to buy more goods. But since the supply of goods is unchanged the increased demand for goods will be met by an increase in prices. This is a typical case of ”more money chasing the same amount of goods.” When the economy is growing, the demand for real balances is also growing. That means that part of the increase in the money supply will not end up chasing goods but rather will end up in the pockets of consumers.

12.6 Balance-of-payments crises

A balance of payments, or BOP, crisis is a situation in which the government is unable or unwilling to meet its financial obligations. These difficulties may manifest themselves in a variety of ways, such as the failure to honor the domestic and/or foreign public debt or the suspension of currency convertibility.

What causes BOP crises? Sometimes a BOP crisis arises as the inevitable consequence of unsustainable combinations of monetary and fiscal policies. A classic example of such a policy mix is a situation in which a government pegs the nominal exchange rate and at the same time runs a fiscal deficit. As we discussed in subsection 12.3, under a fixed exchange rate regime, the government must finance any fiscal deficit by running down its stock of interest bearing assets (see equation (12.11)). Clearly, to the extent that there is a limit to the amount of debt a government is able to issue, this situation cannot continue indefinitely. When the public debt hits its upper limit, the government is forced to change policy. One possibility is that the government stops servicing the debt (i.e., stops paying interest on its outstanding financial obligations), thereby reducing the size of the secondary deficit. This alternative was adopted by Mexico in August of 1982, when it announced that it would be unable to honor its debt commitments according to schedule, marking the beginning of what today is known as the Developing Country Debt Crisis. A second possibility is that the government adopt a fiscal adjustment program by cutting government spending and raising regular taxes and in that way reduce the primary deficit. Finally, the government can abandon the exchange rate peg and resort to monetizing the fiscal deficit. This has been the fate of the vast majority of currency pegs adopted in developing countries. The economic history of Latin America of the past two decades is plagued with such episodes. For example, the currency pegs implemented in Argentina, Chile, and Uruguay in the late 1970s, also known as *tablitas*, ended with large devaluations in the early 1980s; similar outcomes were observed in the Argentine Austral stabilization plan of 1985, the Brazilian Cruzado plan of 1986, the Mexican plan of 1987, and, more recently the Brazilian Real plan of 1994.

An empirical regularity associated with the collapse of fixed exchange rate regimes is that in the days immediately before the peg is abandoned, the central bank looses vast amounts of reserves in a short period of time. The loss of reserves is the consequence of a run by the public against the domestic currency in anticipation of the impending devaluation. The stampede of people trying to massively get rid of domestic currency in exchange for foreign currency is driven by the desire to avoid the loss of real value of domestic currency denominated assets that will take place when the currency is devalued.

The first formal model of the dynamics of a fixed exchange rate collapse is due to Paul R. Krugman of Princeton University.[7] In this section, we will analyze these dynamics using the tools developed in sections 12.3 and 12.4. These tools will helpful in a natural way because, from an analytical point of view, the collapse of a currency peg is indeed a transition from a fixed to a floating exchange rate regime.

Consider a country that is running a constant fiscal deficit $DEF > 0$ each period. Suppose that in period 1 the country embarks in a currency peg. Specifically, assume that the government fixes the nominal exchange rate at E units of domestic currency per unit of foreign currency. Suppose that in period 1, when the currency peg is announced, the government has a positive stock of foreign assets carried over from period 0, $B_0^g > 0$. Further,

[7]The model appeared in Paul R. Krugman, "A Model of Balance-of-Payments Crisis," *Journal of Money, Credit and Banking,* 11, 1979, 311-325.

assume that the government does not have access to credit. That is, the government asset holdings are constrained to being nonnegative, or $B_t^g \geq 0$ for all t. It is clear from our discussion of the sustainability of currency pegs in subsection 12.3 that, as long as the currency peg is in effect, the fiscal deficit produces a continuous drain of assets, which at some point will be completely depleted. Put differently, if the fiscal deficit is not eliminated, at some point the government will be forced to abandon the currency peg and start printing money in order to finance the deficit. Let T denote the period in which, as a result of having run out of reserves, the government abandons the peg and begins to monetize the fiscal deficit.

The dynamics of the currency crisis are characterized by three distinct phases. (1) The pre-collapse phase: during this phase, which lasts from $t = 1$ to $t = T - 2$, the currency peg is in effect. (2) The BOP crisis: It takes place in period $t = T - 1$, and is the period in which the central bank faces a run against the domestic currency, resulting in massive losses of foreign reserves. (3) The post-collapse phase: It encompasses the period from $t = T$ onwards In this phase, the nominal exchange rate floats freely and the central bank expands the money supply at a rate consistent with the monetization of the fiscal deficit.

(1) The pre-crisis phase: from $t = 1$ to $t = T - 2$

From period 1 to period $T - 2$, the exchange rate is pegged, so the variables of interest behave as described in section 12.3. In particular, the nominal exchange rate is constant and equal to E, that is, $E_t = E$ for $t = 1, 2, \ldots, T-2$. By PPP, and given our assumption that $P_t^* = 1$, the domestic price level is also constant over time and equal to E ($P_t = E$ for $t = 1, 2, \ldots, T-2$). Because the exchange rate is fixed, the devaluation rate $(E_t - E_{t-1})/E_{t-1}$, is equal to 0. The nominal interest, i_t, which by the uncovered interest parity condition satisfies $1 + i_t = (1 + r^*)E_{t+1}/E_t$, is equal to r^*. Note that the nominal interest rate in period $T-2$ is also equal to r^* because the exchange rate peg is still in place in period $T-1$. Thus, $i_t = r^*$ for $t = 1, 2, \ldots, T-2$.

As discussed in section 12.3, by pegging the exchange rate the government relinquishes its ability to monetize the deficit. This is because the nominal money supply, M_t, which in equilibrium equals $EL(\bar{C}, r^*)$, is constant, and as a result seignorage revenue, given by $(M_t - M_{t-1})/E$, is nil. Consider now the dynamics of foreign reserves. By equation (12.11),

$$B_t^g - B_{t-1}^g = -DEF; \quad \text{for } t = 1, 2, \ldots, T-2.$$

This expression shows that the fiscal deficit causes the central bank to lose DEF units of foreign reserves per period. The continuous loss of reserves in combination with the lower bound on the central bank's assets, makes it clear that a currency peg is unsustainable in the presence of persistent fiscal imbalances.

(3) The post-crisis phase: from $t = T$ onwards

The government starts period T without any foreign reserves ($B_{T-1}^g = 0$). Given our assumptions that the government cannot borrow (that is, B_t^g cannot be negative) and that it is unable to eliminate the fiscal deficit, it follows that in period T the monetary authority is forced to abandon the currency peg and to print money in order to finance the fiscal deficit. Thus, in the post-crisis phase the government lets the exchange rate float. Consequently, the behavior of all variables of interest is identical to that studied in subsection 12.4. In particular, the government will expand the money supply at a constant rate μ that generates enough seignorage revenue to finance the fiscal deficit. In section 12.4, we deduced that μ is determined by equation (12.15),

$$DEF = L(\bar{C}, i(\mu)) \left(\frac{\mu}{1+\mu} \right)$$

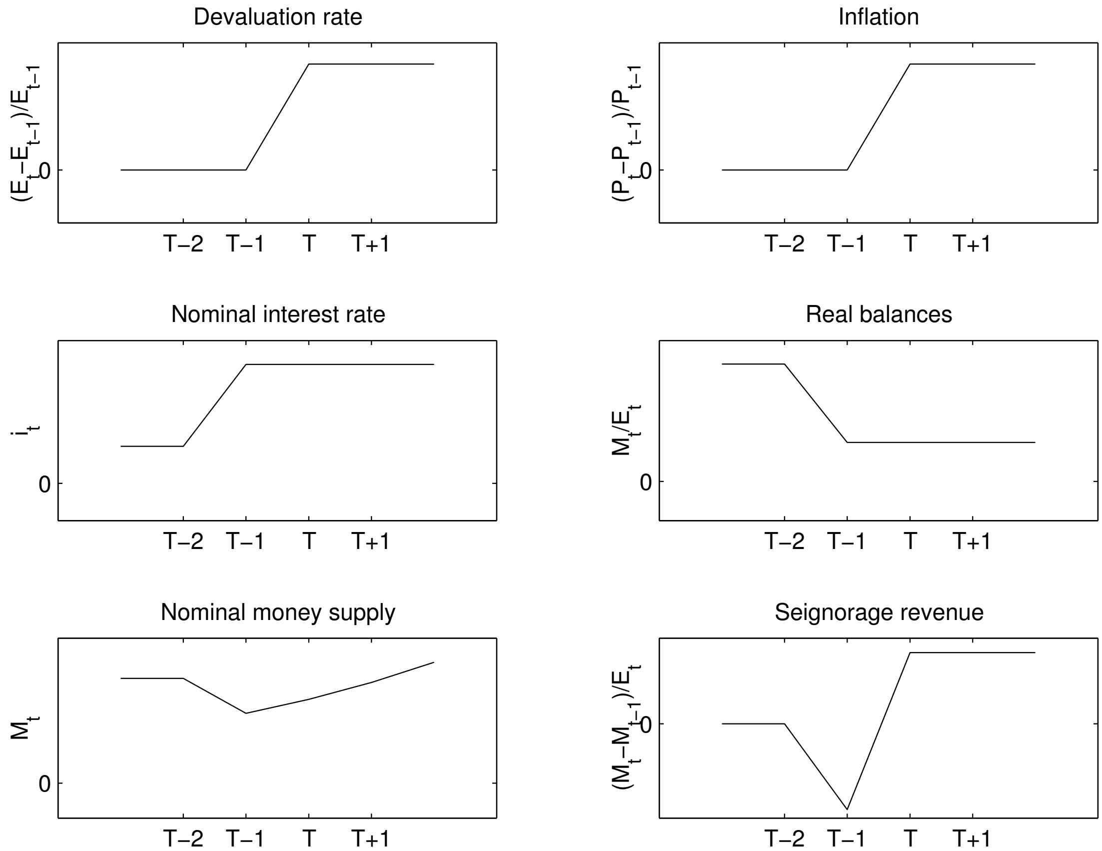
Note that because the fiscal deficit is positive, the money growth rate must also be positive. In the post-crisis phase, real balances, M_t/E_t are constant and equal to $L(\bar{C}, i(\mu))$. Therefore, the nominal exchange rate, E_t, must depreciate at the rate μ. Because in our model $P_t = E_t$, the price level also grows at the rate μ, that is, the inflation rate is positive and equal to μ. Finally, the nominal interest rate satisfies $1 + i_t = (1 + r^*)(1 + \mu)$. Let's compare the economy's pre- and post-crisis behavior. The first thing to note is that with the demise of the fixed exchange rate regime, price level stability disappears as inflation sets in. In the pre-crisis phase, the rate of monetary expansion, the rate of devaluation, and the rate of inflation are all equal to zero. By contrast, in the post-crisis phase these variables are all positive and equal to μ. Second, the sources of deficit finance are very different in each of the two phases. In the pre-crisis phase, the deficit is financed entirely with foreign reserves. As a result, foreign reserves display a steady decline during this phase. On the other hand, in the post-crisis phase the fiscal deficit is financed through seignorage income and foreign reserves are constant (and in our example equal to zero). Finally, in the post-crisis phase real balances are lower than in the pre-crisis phase because the nominal interest rate is higher.

(2) The BOP crisis: period $T-1$

In period $T-1$, the exchange rate peg has not yet collapsed. Thus, the nominal exchange rate and the price level are both equal to E, that is $E_{T-1} = P_{T-1} = E$. However, the nominal interest rate is not r^*, as in the pre-crisis phase, because in period $T-1$ the public expects a depreciation of the domestic currency in period T. The rate of depreciation of the domestic currency between periods $T-1$ and T is μ, that is, $(E_T - E_{T-1})/E_{T-1} = \mu$.[8] Therefore, the nominal interest rate in period $T-1$ jumps up to its post-crisis level $i_{T-1} = (1+r^*)(1+\mu) - 1 = i(\mu)$. As a result of the increase in the nominal interest rate real balances fall in $T-1$ to their post-crisis level, that is, $M_{T-1}/E = L(\bar{C}, i(\mu))$. Because the nominal exchange rate does not change in period $T-1$, the decline in real balances must be brought about entirely through a fall in nominal balances: the public runs to the central bank to exchange domestic currency for foreign reserves. Thus, in period $T-1$ foreign reserves at the central bank fall by more than DEF. To see this more formally, evaluate the government budget constraint (12.10) at

[8] *For technically inclined readers:* To see that $(E_T - E_{T-1})/E_{T-1} = \mu$, use the fact that in $T-1$ real balances are given by $M_{T-1}/E_{T-1} = L(\bar{C}, (1+r^*)E_T/E_{T-1} - 1)$ and that in period T the government budget constraint is $DEF = L(\bar{C}, i(\mu)) - (M_{T-1}/E_{T-1})(E_{T-1}/E_T)$. These are two equations in two unknowns, M_{T-1}/E_{T-1} and E_T/E_{T-1}. If we set $E_T/E_{T-1} = 1+\mu$, then the two equations collapse to (12.15) indicating that $E_T/E_{T-1} = 1+\mu$ and $M_{T-1}/E_{T-1} = L(\bar{C}, i(\mu))$ are indeed the solution.

Figure 12.4: The dynamics of a balance-of-payments crisis



$t = T - 1$ to get

$$\begin{aligned} B^g_{T-1} - B^g_{T-2} &= \frac{M_{T-1} - M_{T-2}}{E} - DEF \\ &= L(\bar{C}, i(\mu)) - L(\bar{C}, r^*) - DEF \\ &< -DEF \end{aligned}$$

The second equality follows from the fact that $M_{T-1}/E = L(\bar{C}, i(\mu))$ and $M_{T-2}/E = L(\bar{C}, r^*)$. The inequality follows from the fact that $i(\mu) =$

$(1+r^*)(1+\mu)-1 > r^*$ and the fact that the liquidity preference function is decreasing in the nominal interest rate. The above expression formalizes Krugman's original insight on why the demise of currency pegs is typically preceeded by a speculative run against the domestic currency and large losses of foreign reserves by the central bank: Even though the exchange rate is pegged in $T-1$, the nominal interest rate rises in anticipation of a devaluation in period T causing a contraction in the demand for real money balances. Because in period $T-1$ the domestic currency is still fully convertible, the central bank must absorb the entire decline in the demand for money by selling foreign reserves. Figure 12.4 closes this section by providing a graphical summary of the dynamics of Krugman-type BOP crises.

12.7 Appendix: A dynamic optimizing model of the demand for money

In this section we develop a dynamic optimizing model underlying the liquidity preference function given in equation (12.6). We motivate a demand money by assuming that money facilitates transactions. We capture the fact that money facilitates transactions by simply assuming that agents derive utility not only from consumption of goods but also from holdings of real balances. Specifically, in each period $t = 1, 2, 3, \ldots$ preferences are described by the following single-period utility function,

$$u(C_t) + z\left(\frac{M_t}{P_t}\right),$$

where C_t denotes the household's consumption in period t and M_t/P_t denotes the household's real money holdings in period t. The functions $u(\cdot)$ and $z(\cdot)$ are strictly increasing and strictly concave functions ($u' > 0$, $z' > 0$, $u'' < 0$, $z'' < 0$).

Households are assumed to be infinitely lived and to care about their entire stream of single-period utilities. However, households discount the future by assigning a greater weight to consumption and real money holdings the closer they are to the present. Specifically, their lifetime utility function is given by

$$\left[u(C_t) + z\left(\frac{M_t}{P_t}\right)\right] + \beta\left[u(C_{t+1}) + z\left(\frac{M_{t+1}}{P_{t+1}}\right)\right] + \beta^2\left[u(C_{t+2}) + z\left(\frac{M_{t+2}}{P_{t+2}}\right)\right] + \ldots$$

Here β is a number greater than zero and less than one called the *subjective discount factor*." The fact that households care more about the present than about the future is reflected in β being less than one.

Let's now analyze the budget constraint of the household. In period t, the household allocates its wealth to purchase consumption goods, P_tC_t, to hold money balances, M_t, to pay taxes, P_tT_t, and to purchase interest bearing foreign bonds, $E_tB_t^p$. Taxes are lump sum and denominated in domestic currency. The foreign bond is denominated in foreign currency. Each unit of foreign bonds costs 1 unit of the foreign currency, so each unit of the foreign bond costs E_t units of domestic currency. Foreign bonds pay the constant world interest rate r^* in foreign currency. Note that because the foreign price level is assumed to be constant, r^* is not only the interest rate in terms of foreign currency but also the interest rate in terms of goods.

That is, r^* is the *real* interest rate.[9] The superscript p in B_t^p, indicates that these are bond holdings of *private* households, to distinguish them from the bond holdings of the government, which we will introduce later. In turn, the household's wealth at the beginning of period t is given by the sum of its money holdings carried over from the previous period, M_{t-1}, bonds purchased in the previous period plus interest, $E_t(1+r^*)B_{t-1}^p$, and income from the sale of its endowment of goods, P_tQ_t, where Q_t denotes the household's endowment of goods in period t. This endowment is assumed to be exogenous, that is, determined outside of the model. The budget constraint of the household in period t is then given by:

$$P_tC_t + M_t + P_tT_t + E_tB_t^p = M_{t-1} + (1+r^*)E_tB_{t-1}^p + P_tQ_t \qquad (12.16)$$

The left hand side of the budget constraint represents the uses of wealth and the right hand side the sources of wealth. The budget constraint is expressed in nominal terms, that is, in terms of units of domestic currency. To express the budget constraint in real terms, that is, in units of goods, we divide both the left and right hand sides of (12.16) by P_t, which yields

$$C_t + \frac{M_t}{P_t} + T_t + \frac{E_t}{P_t}B_t^p = \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + (1+r^*)\frac{E_t}{P_t}B_{t-1}^p + Q_t$$

Note that real balances carried over from period $t-1$, M_{t-1}/P_{t-1}, appear multiplied by P_{t-1}/P_t. In an inflationary environment, P_t is greater than P_{t-1}, so inflation erodes a fraction of the household's real balances. This loss

[9]The domestic nominal and real interest rates will in general not be equal to each other unless domestic inflation is zero. To see this, recall the Fisher equation (8.2). We will return to this point shortly.

of resources due to inflation is called the inflation tax. The higher the rate of inflation, the larger the fraction of their income households must allocate to maintaining a certain level of real balances.

Recalling that P_t equals E_t, we can eliminate P_t from the utility function and the budget constraint to obtain:

$$\left[u(C_t) + z\left(\frac{M_t}{E_t}\right)\right] + \beta\left[u(C_{t+1}) + z\left(\frac{M_{t+1}}{E_{t+1}}\right)\right] + \beta^2\left[u(C_{t+2}) + z\left(\frac{M_{t+2}}{E_{t+2}}\right)\right] + \dots \tag{12.17}$$

$$C_t + \frac{M_t}{E_t} + T_t + B_t^p = \frac{M_{t-1}}{E_t} + (1+r^*)B_{t-1}^p + Q_t \tag{12.18}$$

Households choose C_t, M_t, and B_t^p so as to maximize the utility function (12.17) subject to a series of budget constraints like (12.18), one for each period, taking as given the time paths of E_t, T_t, and Q_t. In choosing streams of consumption, money balances, and bonds, the households faces two tradeoffs. The first tradeoff is between consuming today and saving today to finance future consumption. The second tradeoff is between consuming today and holding money today.

Consider first the tradeoff between consuming one extra unit of the good today and investing it in international bonds to consume the proceeds tomorrow. If the household chooses to consume the extra unit of goods today, then its utility increases by $u'(C_t)$. Alternatively, the household could sell the unit of good for 1 unit of foreign currency and with the proceeds buy 1 unit of the foreign bond. In period $t + 1$, the bond pays $1 + r^*$ units of foreign currency, with which the household can buy $(1 + r^*)$ units of goods.

This amount of goods increases utility in period $t+1$ by $(1+r^*)u'(C_{t+1})$. Because households discount future utility at the rate β, from the point of view of period t, lifetime utility increases by $\beta(1+r^*)u'(C_{t+1})$. If the first alternative yields more utility than the second, the household will increase consumption in period t, and lower consumption in period $t+1$. This will tend to eliminate the difference between the two alternatives because it will lower $u'(C_t)$ and increase $u'(C_{t+1})$ (recall that $u(\cdot)$ is concave, so that $u'(\cdot)$ is decreasing). On the other hand, if the second alternative yields more utility than the first, the household will increase consumption in period $t+1$ and decrease consumption in period t. An optimum occurs at a point where the household cannot increase utility further by shifting consumption across time, that is, at an optimum the household is, in the margin, indifferent between consuming an extra unit of good today or saving it and consuming the proceeds the next period. Formally, the optimal allocation of consumption across time satisfies

$$u'(C_t) = \beta(1+r^*)u'(C_{t+1}) \tag{12.19}$$

We will assume for simplicity that the subjective rate of discount equals the world interest rate, that is,

$$\beta(1+r^*) = 1 \tag{12.20}$$

Combining this equation with the optimality condition (12.19) yields,

$$u'(C_t) = u'(C_{t+1}) \tag{12.21}$$

Because $u(\cdot)$ is strictly concave, $u'(\cdot)$ is monotonically decreasing, so this expressions implies that $C_t = C_{t+1}$. This relationship must hold in all periods, implying that consumption is constant over time. Let $\bar{C}$ be this optimal level of consumption. Then, we have

$$C_t = C_{t+1} = C_{t+2} = \cdots = \bar{C}$$

Consider now the tradeoff between spending one unit of money on consumption and holding it for one period. If the household chooses to spend the unit of money on consumption, it can purchase $1/E_t$ units of goods, which yield $u'(C_t)/E_t$ units of utility. If instead the household chooses to keep the unit of money for one period, then its utility in period t increases by $z'(M_t/E_t)/E_t$. In period $t+1$, the household can use the unit of money to purchase $1/E_{t+1}$ units of goods, which provide $u'(C_{t+1})/E_{t+1}$ extra utils. Thus, the alternative of keeping the unit of money for one period yields $z'(M_t/E_t)/E_t + \beta u'(C_{t+1})/E_{t+1}$ additional units of utility. In an optimum, the household must be indifferent between keeping the extra unit of money for one period and spending it on current consumption, that is,

$$\frac{z'(M_t/E_t)}{E_t} + \beta \frac{u'(C_{t+1})}{E_{t+1}} = \frac{u'(C_t)}{E_t} \tag{12.22}$$

Using the facts that $u'(C_t) = u'(C_{t+1}) = u'(\bar{C})$ and that $\beta = 1/(1+r^*)$ and rearranging terms we have

$$z'\left(\frac{M_t}{E_t}\right) = u'(\bar{C})\left[1 - \frac{E_t}{(1+r^*)E_{t+1}}\right] \tag{12.23}$$

Using the uncovered interest parity condition (12.8) we can write

$$z'\left(\frac{M_t}{E_t}\right) = u'(\bar{C})\left(\frac{i_t}{1+i_t}\right) \tag{12.24}$$

This equation relates the demand for real money balances, M_t/E_t, to the level of consumption and the domestic nominal interest rate. Inspecting equation (12.24) and recalling that both u and z are strictly concave, reveals that the demand for real balances, M_t/E_t, is decreasing in the level of the nominal interest rate, i_t, and increasing in consumption, $\bar{C}$. This relationship is called the *liquidity preference function.* We write it in a compact form as

$$\frac{M_t}{E_t} = L(\bar{C}, i_t)$$

which is precisely equation (12.6).

The following example derives the liquidity preference function for a particular functional form of the period utility function. Assume that

$$u(C_t) + z(M_t/E_t) = \ln C_t + \gamma \ln(M_t/E_t).$$

Then we have $u'(\bar{C}) = 1/\bar{C}$ and $z'(M_t/E_t) = \gamma/(M_t/E_t)$. Therefore, equation (12.24) becomes

$$\frac{\gamma}{M_t/E_t} = \frac{1}{\bar{C}}\left(\frac{i_t}{1+i_t}\right)$$

The liquidity preference function can be found by solving this expression for M_t/E_t. The resulting expression is in fact the liquidity preference function given in equation (12.5.2), which we reproduce here for convenience.

$$\frac{M_t}{E_t} = \gamma \bar{C} \left(\frac{i_t}{1 + i_t} \right)^{-1}$$

In this expression, M_t/E_t is linear and increasing in consumption and decreasing in i_t.